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Nulling-cancelling algorithm with selective maximum-likelihood detection for MIMO communication

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Nulling-cancelling algorithm with selective maximum-likelihood detection for MIMO communication

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ABSTRACT

Multiple transmit and receive antennas can increase the system capacity, as well as increase reliability in wireless communication. Vertical Bell Laboratories layered spaces-time (V-BLAST) scheme is widely used to achieve high spectral efficiencies in scattering environments. In V-BLAST systems, receiver design is usually based on the nulling-canceling algorithm which offers a good tradeoff between the computational complexity and system performance.

In this thesis, we propose a nulling-canceling based detection algorithm that performs selective maximum-likelihood decoding. We first compare the symbol estimates from two nulling-canceling implementations with different orders. If the symbol estimates do not agree, then maximum-likelihood detection is performed on the discrepant symbols and the rest of the symbols are detected via nulling and canceling. If there is no discrepancy in the comparison, then only nulling and canceling are performed. In our numerical results, 4-QAM (Quadrature Amplitude Modulation) and 16-QAM constellations are considered, and both Minimum Mean Squared Error (MMSE) and Zero-Forcing (ZF) based detections are implemented. We show that our proposed algorithm can achieve a better performance than the nulling-canceling algorithm and requires a relatively small increase in computational complexity, especially at high SNR.

Based on the Bit Error Rate (BER) performance result, we show that our proposed algorithm can achieve a better performance than the nulling-canceling algorithm and requires a relatively small computational complexity increase, especially at high Signal-to-Noise Ratio (SNR) scenario. The BER performances of an unordered system with BPSK (Binary Phase Shift Keying) or 4-QAM modulation and hybrid detection algorithms are given, under the joint consideration of nulling-canceling of several subchannels and block maximum-likelihood

detection of several subchannels.

CHAPTER 1. INTRODUCTION

1.1 Literature Review

Reliability and high data rate are always the most important concerns in wireless communication technique. In recent years, using multiple antennas at transmitter and/or receiver has emerged as one of the most promising approaches for high data rate wireless transmission. Multiple-element antennas can improve the performance and capacity of a wireless communication system in a fading environment, and shows a higher spectral efficiency than the conventional communication systems [1, 2, 3]. To obtain full diversity order for a reliable transmission, serial encoding can be used to transmit an encoded bit stream over all the transmit antennas. Codeword is made through serial encoding and then is interleaved and mapped to a constellation point, before demultiplexing onto different antennas. For a MIMO system with M transmit antennas and N receive antennas, at each time M symbols in the codeword are transmitted consequently by the M transmit antennas.

Bell Laboratories Layered Space Time (BLAST) architecture is an innovative work in achieving reliable transmission [4, 5]. The original architecture is given in [6], which is called diagonal Bell Laboratories layered space-time (D-BLAST), is capable of approaching the Shannon capacity for multiple transmitters and receivers, but is complex to implement. A simplified version, vertical BLAST (V-BLAST) [7] is considered as a parallel encoding technique which splits the information bit stream into several substreams and transmit them in parallel using a set of transmit antennas at the same time and frequency. At the receiver side, each stream is detected by a sequential nulling and canceling scheme. We can null out the interferes by weighting the received signal vectors with a zero-forcing (ZF) or minimum mean squared error (MMSE) nulling vector, where the subchannel signal with the highest SNR is detected first. Its contribution is subtracted from the received signal. The strongest remaining transmit signal is decoded then, and so on. This system comes with a simple decoding complexity that is linear in the number of antennas, where the propagation of errors from one step of detection to the next step is minimized, but requires a multiple calculation of pseudo-inverses [10]. Optimal decoding can be obtained with the joint detection of the transmitted codewords, such as maximum-likelihood (ML) decoding. However, the extreme complexity of ML decoding generally precludes its use in practical multi-antenna systems, especially when large signal constellations or many transmit antennas are involved. A method is proposed in [11] where the receiver complexity can be significantly reduced by using of symbol interference cancelation, which avoids the joint decoding procedure and maintains a relatively satisfying performance.

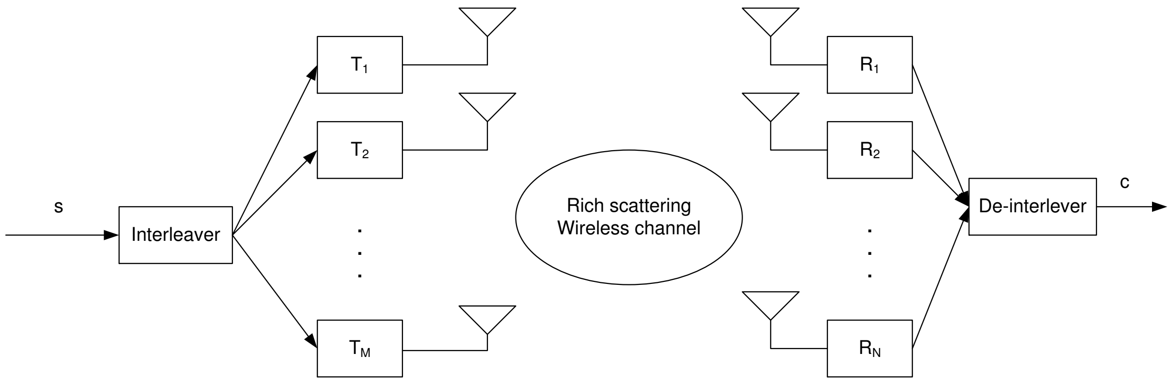

Figure 1.1 V-BLAST system model

Fig. 1.1 shows the block diagram of the V-BLAST system, where s is the signal before interleaver and mapping to the constellation model, T_1 to T_M are the transmit antennas and R_1 to R_N are the receive antennas. c is the received signal after de-interleaver and decoding.

Several methods for detecting the transmitted symbols, including the nulling and canceling algorithm and its variants [7, 12, 13] have been reported. In [14], an algorithm is proposed which detects a number of probable streams simultaneously based on the first detected substream, and then chooses the most probable stream among them. In [13], a combined ML and decision feedback (DFE) decoding scheme is proposed, which performs block ML detection on a number of substreams first, then use DFE to cancel their interferences. The method can achieve a good performance on error probability but block ML detection increases the computational complexity. In [15], a method that avoids the calculation of pseudo-inverse matrix is proposed, which reduces the computation complexity from $\mathcal{O}(M^4)$ to $\mathcal{O}(M^3)$. In [16], a nulling-canceling based decoding algorithm for coded MIMO systems is proposed. A modified square-root algorithm with SNR ordering and soft interference cancelation is used to achieve a near-optimal performance result.

1.2 Thesis Contribution

In this thesis, we propose a MIMO detection algorithm that combines the nulling and canceling algorithm and block maximum-likelihood detection algorithm together to achieve an improved performance. The algorithm relies on comparing the detection results of two nulling-canceling algorithms with different orderings. Block maximum likelihood estimation will be performed when comparison does not achieve agreement. In our proposed algorithm, system diversity order can be increased by performing maximum-likelihood detection on more symbols, and average error probability can be reduced by this method compared with the original nulling-canceling algorithm.

We introduce an empirical parameter ϵ to quantify the percentage of block ML detection in our proposed algorithms. We also perform the complexity analysis of our proposed system, and provide a theoretical performance of an unordered system with BPSK and 4-QAM modulation.

1.3 Thesis Outline

The remaining part of this thesis is organized as follows. In Chapter 2, we briefly introduce the MIMO system model under a rich-scattering Rayleigh distributed channel condition. Three decoding algorithms based on zero-forcing, minimum-mean-square-error and maximum-likelihood are reviewed. The pros and cons of the decoding algorithms are given. Also, a detailed description of nulling-canceling algorithm is introduced. SNR and LLR based ordering schemes are introduced in this section. In Chapter 3, the motivation of our proposed algorithm is presented. Then we describe our proposed selective maximum-likelihood detection method based on SNR ordering and MMSE or ZF-based filtering. Chapter 4 evaluates the theoretical complexity and the performance of our proposed selective maximum-likelihood algorithm is analyzed in Chapter 4. Numerical simulation results of the error probability and complexity are provided in Chapter 5. A computational complexity comparison is included with comparison of some other algorithms. We make the conclusions and present some possible future work Chapter 6.

1.4 Notations

Bold faced letters denote random variables, vectors or matrices; plain letters denote the corresponding realizations or constant. In this thesis, $(\mathbf{x})^{\dagger}$ denotes Moore-Penrose pseudo-inverse of matrix $\mathbf{x}$; $(\mathbf{x})^{*}$ denotes conjugate transpose (Hermitian) of matrix $\mathbf{x}$; $(\mathbf{x})^{T}$ denotes the matrix transpose of matrix $\mathbf{x}$. $\mathbf{I}_m$ denotes $m \times m$ identity matrix; $\parallel \mathbf{x} \parallel$ denotes the Euclidean norm of the vector $\mathbf{x}$; $\parallel \mathbf{x} \parallel_2$ denotes the matrix row norm (Euclidean norm calculated along the row direction of the matrix); $\Pr(x)$ denotes the probability of event x; $\mathrm{Cov}(\mathbf{x})$ denotes the covariance matrix of matrix $\mathbf{x}$; $\mathrm{Re}(x)$ denotes x as a real number, vector or matrix; $\mathrm{Im}(x)$ denotes x as a imaginary number, vector or matrix; $\lfloor x \rfloor$ finds the largest integer less than or equal to x; $\lceil x \rceil$ finds the smallest integer grater than or equal to x.

CHAPTER 2. SYSTEM MODEL AND NULLING-CANCELING ALGORITHM

2.1 System Model

We consider an uncoded rich-scattering MIMO system with M transmit antennas and N receive antennas. We assume that $N \geq M$. The system input-output relationship can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2.1}$$

where $\mathbf{H}$ is the $N \times M$ channel matrix, $\mathbf{x} = [x_1, \ldots, x_M]^T$ is the transmitted signal, $\mathbf{y} = [y_1, \ldots, y_N]^T$ is the received signal, and $\mathbf{n} = [n_1, \ldots, n_N]^T$ is the circularly symmetric complex additive white Gaussian noise (AWGN) with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_N$. The entries of $\mathbf{x}$ are chosen independently from L-ary quadrature amplitude modulation (QAM) constellation ($L = 4$ or 16 in this thesis), with second moment equals to the signal to noise ratio (SNR) ρ. It is assumed that the same constellation is employed for all the subchannels. All entries of $\mathbf{H}$ are independent and identically distributed (iid) complex Gaussian random variables with zero mean and unit variance. Therefore, the channel $\mathbf{H}$ is a Rayleigh fading channel. The signals are narrow-band, and hence, the channel can be considered as frequency flat channel. We assume that the channel $\mathbf{H}$ is known perfectly at the receiver.

A Type III QAM constellation diagram [17, 18] is used in our simulation. Fig. 2.1 shows the Type III 16-QAM constellation diagram.

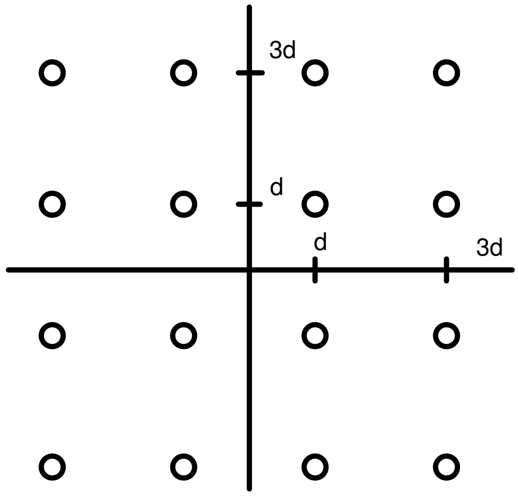

Figure 2.1 Type III 16-QAM constellation

2.2 Existing Decoding Algorithms

In this section we recall several commonly used detection methods with respect to the ZF, MMSE and maximum-likelihood (ML) criterion, sphere-decoding and nulling cancelling algorithms. In a linear detector (ZF and MMSE), the receiver $\mathbf{y}$ is multiplied with a pseudo-inverse filter matrix $\mathbf{G}$, where $\mathbf{G}$ is decided by the channel matrix. Then a parallel decision estimation is used on the layers to decode the transmitted symbols. Besides, there are also some other MIMO detectors that are used as the decoding algorithms. A "list" version of the sphere decoder is proposed in [19] on the coded systems to achieve a superior performance, but comes with more complexity than MMSE. Nulling and canceling algorithm is the major detection algorithm used in BLAST scheme.

2.2.1 Zero Forcing

2.2.1.1 Zero forcing detector

Intersymbol interference (ISI) is a signal distortion that causes the previously transmitted symbol to have an effect on the currently received symbol, which makes the communication process unreliable. A way to mitigate the influence of ISI is using a zero-forcing detector.

In a zero-forcing detector, the mutual interference between different receive antennas will be perfectly suppressed. The complexity of the zero-forcing algorithm is in the cubic order. From the Moore-Penrose pseudo-inverse of the channel matrix, we have:

$$\mathbf{G}_{ZF} = \mathbf{H}^{\dagger} = (\mathbf{H}^{*}\mathbf{H})^{-1}\mathbf{H}^{*} \tag{2.2}$$

where $\mathbf{H}$ has full column rank.

Then, left multiply $\mathbf{G}_{ZF}$ on the receiver side, where we get the zero-forcing detector:

$$\tilde{\mathbf{x}}_{ZF} = \mathbf{G}_{ZF}\mathbf{y} = \mathbf{H}^{\dagger}\mathbf{y} = \mathbf{x} + (\mathbf{H}^{*}\mathbf{H})^{-1}\mathbf{H}^{*}\mathbf{n}. \tag{2.3}$$

Then we map each element of the filter output vector $\tilde{\mathbf{x}}_{ZF}$ onto an element $\mathbf{x}$ of the symbol alphabet $\mathbf{X}$ with a minimum distance criterion and find the estimation result, where we have:

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbf{X}} \parallel \mathbf{x} - \tilde{\mathbf{x}}_{ZF} \parallel \tag{2.4}$$

The estimation errors of different layers correspond to the main diagonal elements of the error covariance matrix which equals the noise covariance matrix after the receive filter. The zero-forcing detector can always remove the ISI and is ideal when the channel is noise free. However, consider the small eigenvalue of $\mathbf{H}^{*}\mathbf{H}$ which corresponds to a small magnitude of the channel response in the frequency domain, the effect of noise is significantly amplified. It can be shown [20] that when $M = N \to \infty$, noise amplification tends to infinity as well. Therefore, the effect of the noise term should also be considered in the design of the filter matrix $\mathbf{G}$.

2.2.1.2 Zero forcing BLAST

The zero-forcing based BLAST interference cancelation was first proposed in [7] where signals are detected one by one at the receiver side, assuming that the subchannel i comes with the largest SNR (the smallest estimation error) after interference nulling. Denote $\mathbf{g}_{ZF}^{(i)}$ as the ith row of $\mathbf{G}_{ZF}$ in (2.2), and $\lambda_i = \mathbf{g}_{ZF}^{(i)}n$ as the effective noise to find the estimation of x_i. The first step of ZF-based BLAST detection can be made by

$$\tilde{x}_i = \mathbf{g}_{ZF}^{(i)}\mathbf{y} = \mathbf{g}_{ZF}^{(i)}(\mathbf{H}\mathbf{x} + \mathbf{n}) = x_i + \lambda_i \tag{2.5}$$

$\hat{x}_i$ can be found from the mapping of $\tilde{x}_i$ to an element of the symbol alphabet with a minimum distance criterion. The interference of this signal is subtracted from the received signal $\mathbf{y}$, with the removal of the ith column from the channel matrix. We then obtain the modified receive signals (where y_1 is the receive signals after subtraction of $\hat{x}_i$)

$$\mathbf{y_1} = \mathbf{y} - \mathbf{h}_i \hat{x}_i \tag{2.6}$$

and the reduced order channel matrix

$$\mathbf{H}_1 = [\mathbf{h}_1, \ldots, \mathbf{h}_{i-1}, \mathbf{h}_{i+1}, \ldots, \mathbf{h}_M] \tag{2.7}$$

The reduced order system comes with $M-1$ transmit antennas. We continue find and remove the interference with the largest SNR, estimate the corresponding transmit signal until all the signals are successfully detected.

2.2.2 Minimum Mean Squared Error

2.2.2.1 MMSE detector

The MMSE detector minimizes the mean squared error (MSE) between the output of the linear detector and the actually transmitted symbols. The result is presented in [21] where filter matrix of MMSE can be written as:

$$\mathbf{G}_{MMSE} = (\mathbf{H}^*\mathbf{H} + \sigma_n^2 \mathbf{I}_M)^{-1}\mathbf{H}^*. \tag{2.8}$$

The resulting filter output is given by:

$$\tilde{\mathbf{x}}_{MMSE} = \mathbf{G}_{MMSE}\mathbf{y} = (\mathbf{H}^*\mathbf{H} + \sigma_n^2 \mathbf{I}_M)^{-1}\mathbf{H}^*(\mathbf{H}\mathbf{x} + \mathbf{n}). \tag{2.9}$$

The estimation errors of the different layers correspond to the main diagonal elements of the error covariance matrix

$$\mathbf{\Phi}_{MMSE} = E\{(\tilde{\mathbf{x}}_{MMSE} - \mathbf{x})(\tilde{\mathbf{x}}_{MMSE} - \mathbf{x})^*\} = \sigma_n^2(\mathbf{H}^*\mathbf{H} + \sigma_n^2 \mathbf{I}_M)^{-1}. \tag{2.10}$$

Define an extended $(M+N) \times M$ channel matrix $\bar{\mathbf{H}}$ and an extended $(M+N) \times 1$ receive vector $\bar{\mathbf{y}}$ where

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_M \end{bmatrix} \tag{2.11}$$

and

$$\bar{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ 0_{M,1} \end{bmatrix}. \tag{2.12}$$

Therefore, the MMSE filter can be rewritten as

$$\tilde{\mathbf{x}}_{MMSE} = (\bar{\mathbf{H}}^* \bar{\mathbf{H}})^{-1} \bar{\mathbf{H}}^* \bar{\mathbf{y}} = \bar{\mathbf{H}}^{\dagger} \bar{\mathbf{y}} \tag{2.13}$$

and the error covariance matrix becomes

$$\mathbf{\Phi}_{MMSE} = \sigma_n^2 (\bar{\mathbf{H}}^* \bar{\mathbf{H}})^{-1}. \tag{2.14}$$

Compare the equations in zero-forcing and MMSE, we can find that (2.13) and (2.14) have the same form of the expressions as (2.9) and (2.10), only with the change from $\mathbf{H}$ to $\bar{\mathbf{H}}$.

MMSE detector may not eliminate the ISI completely but can minimize the total power of the noise components in the output. MMSE is considered as a good tradeoff between interference suppression and noise amplification. Even though, MMSE algorithm achieves a good performance compared to ZF algorithm. There is a significant gap between its performance and that of the maximum-likelihood algorithm. Besides, the MMSE performance will degrade significantly if channel matrix is rank deficient.

2.2.2.2 MMSE BLAST

The MMSE BLAST can be derived with the same steps as ZF BLAST, implementing the filter matrix in equation (2.8). Let $\lambda_i = \mathbf{g}_{MMSE}^{(i)} \mathbf{n}$ as the effective noise to find the estimation of x_i, and $\mathbf{g}_{MMSE}^{(i)}$ as the ith row of $\mathbf{G}_{MMSE}$ in equation (2.8). The first step of MMSE-basd BLAST detection can be expressed by

$$\tilde{x}_i = \mathbf{g}_{MMSE}^{(i)} \mathbf{y} = \mathbf{g}_{MMSE}^{(i)} (\mathbf{Hx} + \mathbf{n}) = x_i + \lambda_i \tag{2.15}$$

Then, following the same steps in equation (2.6) and (2.7), we can detect all the symbols.

2.2.3 Maximum Likelihood

For a given set of data and the probability model, maximum-likelihood picks the value from the model parameters, that make the data most probable [14]. Consider a family D_θ

of probability distributions parameterized by an unknown parameter θ, associated with a probability density function f_θ (for continues distribution). A set of n entries $z_1, z_2, \ldots, z_n$ is chosen from the distribution. The likelihood function with respect to θ is:

$$L(\theta) = f_\theta(z_1, \ldots, z_n|\theta).$$

By finding the value of θ that maximizes $L(\theta)$, the maximum-likelihood estimation of θ can be defined as:

$$\hat{\theta} = \arg\max_{\theta} \mathbf{L}(\theta).$$

The maximum-likelihood decoding in our system model requires the minimization of the metric

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbf{X}} \parallel \mathbf{y} - \mathbf{H}\mathbf{x} \parallel \tag{2.16}$$

over all the possible points of the lattice in the set of $\mathbf{X}$ where $\mathbf{X}$ is the set of all possible positions in the selected modulation scheme. Block ML detection can be applied on the selective symbols detection with the minimization of the combined norm of the metric.

Block ML detection will achieve a best error probability performance among the three algorithms (ZF, MMSE and ML) but requires a significant complexity increase especially when the selected columns number is large.

2.2.4 Sphere Decoding Algorithm

Sphere decoding algorithm was first presented in [8], and introduced for space-time decoding in [9]. The algorithm is originally designed for a system where transmit signal $\mathbf{x}$ is choosing from a real lattice. The sphere decoding algorithm can also be used on complex lattices as a complex sphere decoder. The performance of the sphere decoding relates with the choice of an initial radius r, and can be extended to a complex system as well only when the real and imaginary components of $\mathbf{y}$, $\mathbf{H}$ and $\mathbf{x}$ can be decoupled and create a system of real equations with twice the dimension of the original system [19].

A complete review of sphere decoding was presented in [19]. For a real constellation and channels, given channel knowledge, $\mathbf{s}_i$ as an $V \times 1$ vector of transmit data bits, $\mathbf{x}$ is a vector after mapping from $\mathbf{s}$, where x_i is the mapping from $\mathbf{s}_i, i = 1, \ldots, M$. Under the same system model in (2.1), the sphere decoder solves the problem

$$\min_{\mathbf{x} \in \mathbf{\Lambda}} (\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{H}^T \mathbf{H} (\mathbf{x} - \hat{\mathbf{x}}) \tag{2.17}$$

where $\hat{\mathbf{x}}$ denotes the search sphere center, $\mathbf{\Lambda}$ is a lattice where each entry of the M-dimensional vector $\mathbf{x}$ is taken from a constellation of 2^V consecutive integers.

$$\hat{\mathbf{x}} = (\mathbf{H^T H})^{-\mathbf{1}} \mathbf{H^T y} \tag{2.18}$$

is the unconstrained ML estimate of $\mathbf{x}$.

Unless $\mathbf{H}$ has orthogonal columns, by which the M-dimensional search can be simplified to M of one-dimensional search. In other case, the search needs to study 2^{MV} hypotheses. The sphere decoder provides a reduced complexity way to conduct the search lie inside a sphere

$$(\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{H}^T \mathbf{H} (\mathbf{x} - \hat{\mathbf{x}}) \leq r^2 \tag{2.19}$$

where r is the radius that contains the solution of $\mathbf{x}$, $r > 0$.

Since $\mathbf{H}^T\mathbf{H}$ in (2.19) is a symmetric positive-definite matrix, which can be decomposed into an upper triangular matrix and the transpose of this upper triangular matrix (also named as Cholesky decomposition). Denote $\mathbf{U}$ as an $M \times M$ upper triangular matrix where $\mathbf{U}^T\mathbf{U} = \mathbf{H}^T\mathbf{H}$. Denote the entries of $\mathbf{U}$ as u_{ij}, $i \leq j = 1, \ldots, M$, $u_{ii} > 0$. Therefore, equation (2.19) can be written as

$$(\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{U}^T \mathbf{U} (\mathbf{x} - \hat{\mathbf{x}}) = \sum_{i=1}^{M} u_{ii}^2 \left[x_i - \hat{x}_i + \sum_{j=i+1}^{M} \frac{u_{ij}}{u_{ii}} (x_j - \hat{x}_j) \right]^2 \leq r^2 \tag{2.20}$$

The sphere decoder makes the joint decision from x_M to x_1 by using the inequality of equation (2.20), where we set $i = M$ first, obtain that

$$u_{MM}^2 (x_M - \hat{x}_M) \leq r^2 \tag{2.21}$$

or equivalently, we have

$$\lceil \hat{x}_M - \frac{r}{u_{MM}} \rceil \leq x_M \leq \lfloor \hat{x}_M + \frac{r}{u_{MM}} \rfloor \tag{2.22}$$

The sphere decoder chooses a candidate value of x_M from the range in (2.22), and continue computes the x_{M-1} from inequality (2.20)

$$u_{M-1,M-1}^2 \left[x_{M-1} - \hat{x}_{M-1} + \frac{u_{M-1,M}}{u_M M}(x_M - \hat{x}_M)\right]^2 + u_{MM}^2 (x_M - \hat{x}_M)^2 \leq r^2 \tag{2.23}$$

which yields the lower bound

$$x_{M-1} \geq \lceil \hat{x}_{M-1} - \frac{\sqrt{r^2 - u_{MM}^2 (x_M - \hat{x}_M)^2}}{u_{M-1,M-1}} + \frac{u_{M-1,M}}{u_M M}(x_M - \hat{x}_M) \rceil \tag{2.24}$$

The corresponding upper bound can also be derived. A $\hat{x}_{M-1}$ is chosen by the sphere decoding algorithm in between the range of the upper and lower bounds, and proceeds to x_{M-2}, and so on.

There may be two things happen in the end. Either the decoder successfully reaches x_1 and $\mathbf{x}$ is successfully chosen from the computed range. Or the decoder can not find any point in between the range of the upper and lower bounds for a specific symbol x_m. For the first case, the whole set of the candidates of $\mathbf{x}$ can be used to recalculate the radius r, then a refined search of $\mathbf{x}$ with smaller r is processed and a better estimation result may achieve. For the second case, at least one bad candidate choice has been made for $x_{m+1}, \ldots, x_M$. In that case, the decoder revise the x_{m+1} by choosing another candidate then recalculate the bounds for x_m. When no more available candidates can be chosen for x_{m+1}, it goes back to choose a new candidate for x_{m+2}, and so on.

The performance of the sphere decoding algorithm is relevant with the choice of r. The bigger the r is chosen, the longer time the search takes. In the opposite, when r is quite small, the algorithm may not find appropriate candidate points between the range of the bounds.

2.2.5 Nulling-Canceling Algorithm

Nulling-canceling algorithm is an existing decoding algorithm based on the BLAST detection algorithm. In nulling-canceling algorithm, interference nulling can be considered as a feedforward filter, and interference canceling works like a feedback filter.

The nulling-canceling algorithm includes interference nulling, interference canceling and ordering. In practice, the algorithm proceeds in the order of ordering, nulling and cancelation.

2.2.5.1 Ordering

In this section we recall the linear detection with respect to the ZF and to the MMSE criterion. The review is based on [12].

Given the received signal $\mathbf{y}$, the linear minimum mean squared error (LMMSE) estimation of $\mathbf{x}$ is

$$\tilde{\mathbf{x}} = (\omega I + \mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{x} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_M \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} \tag{2.25}$$

where $\omega = (\sigma_n^2)$. Denoting $\mathbf{H}_\omega^\dagger$ as the first N columns of the pseudo-inverse, so the i-th row of $\mathbf{H}_\omega^\dagger$ is $\mathbf{H}_{\omega,i}^\dagger$, we have $\tilde{\mathbf{x}} = \mathbf{H}_\omega^\dagger \mathbf{y}$ and $\tilde{x}_i = \mathbf{H}_{\omega,i}^\dagger \mathbf{y}$. $\mathbf{H}_{\omega,i}$ is the MMSE nulling vector, and $\tilde{\mathbf{x}}_i$ is the MMSE estimate of the ith symbol. The covariance matrix of the estimation error $\mathbf{x} - \tilde{\mathbf{x}}$ can be written as

$$E(\mathbf{x} - \tilde{\mathbf{x}})(\mathbf{x} - \tilde{\mathbf{x}})^* = (\omega \mathbf{I} + \mathbf{H}^*\mathbf{H})^{-1} := \mathbf{\Sigma}. \tag{2.26}$$

$\mathbf{\Sigma}$ is denoted as the covariance matrix of $\mathbf{x}$.

Zero-forcing (ZF) based nulling-canceling algorithm can be obtained by setting ω to zero in (2.25), where

$$\tilde{\mathbf{x}} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{x} = \begin{bmatrix} \mathbf{H} \\ \mathbf{0} \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} \tag{2.27}$$

and

$$E(\mathbf{x} - \tilde{\mathbf{x}})(\mathbf{x} - \tilde{\mathbf{x}})^* = (\mathbf{H}^*\mathbf{H})^{-1} := \mathbf{\Sigma}. \tag{2.28}$$

For nulling-canceling detection algorithm, the order in which the components of $\mathbf{x}$ are detected and canceled is important to the overall system performance. A Log-Likelihood Ratio (LLR) based nulling-canceling scheme is introduced in [22] which uses *a posteriori* probabilities (APP) to compute and cancel the soft interferences. The APP is expressed in the form of a log-probability ratio (LPR). Maximizing the APP of a given symbol will minimize the error probability on that symbol.

The ordering determined by the order of the diagonal entry of $\mathbf{\Sigma}$ ((2.25),(2.27)) is called "SNR ordering" in nulling-canceling scheme. The minimum diagonal entry of $\mathbf{\Sigma}$ corresponds to the transmitted symbol with the largest post-estimation SNR and the worst channel statistic.

The symbol corresponding to that largest SNR is usually detected first. Its contribution is subtracted from the received signal. The strongest remaining transmit signal is decoded then, and so on.

2.2.5.2 Nulling and canceling

In this thesis, we focus on the nulling-canceling algorithm with the SNR ordering technique.

In SNR ordering, the symbols that have been detected will have their effects removed from the received signal, assuming the decisions are correct. The process continues until all symbols are detected.

It has been shown in several publications that the decoding process of V-BLAST can be expressed in terms of the QR decomposition of the channel matrix H [23, 24] . In the QR factorization process, $\mathbf{H} = \mathbf{QR}$ where $\mathbf{Q}$ is an $N \times M$ orthogonal matrix such that $\mathbf{Q}^*\mathbf{Q} = \mathbf{I_M}$ and $\mathbf{R}$ is an $M \times M$ upper triangular matrix. The amplitudes of the entries of the matrix R are known to be χ-distributed with different degrees of freedom.

Left multiply $\mathbf{Q}^*$ on each side of (2.1), we get the unsorted BLAST receiver:

$$\tilde{\mathbf{y}} = \mathbf{Q}^*\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{Q}^*\mathbf{n} \tag{2.29}$$

where $\tilde{\mathbf{y}}$ is denoted as the unsorted receiver after multiplication of $\mathbf{Q}^*$. Note that since $\mathbf{Q}$ is unitary, the statistical properties of the noise vector $\mathbf{Q}^*\mathbf{n}$ are the same as those of $\mathbf{n}$. Denote $\tilde{\mathbf{n}} = \mathbf{Q}^*\mathbf{n}$, we have:

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \dots \\ \tilde{y}_M \end{bmatrix} = \begin{bmatrix} R_{1,1} & \dots & R_{1,M-1} & R_{1,M} \\ \dots & \dots & \dots & \dots \\ 0 & \dots & R_{M-1,M-1} & R_{M-1,M} \\ 0 & \dots & 0 & R_{M,M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_M \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \dots \\ \tilde{n}_M \end{bmatrix}. \tag{2.30}$$

Denote $\hat{\mathbf{x}}$ as the hard decision of $\bar{\mathbf{x}}$, where $\bar{x}_M$ can be found by: $\bar{x}_M = \frac{\tilde{y}_M}{R_{M,M}}$. Using the recursion algorithm, $\bar{x}_{M-i}$ (i from 0 to $\alpha - 1$) can be decided by:

$$\bar{x}_{M-i} = \frac{\tilde{y}_{M-i} - \sum_{j=0}^{i-1} R_{M-i,M-j}\hat{x}_{M-j}}{R_{M-i,M-i}}. \tag{2.31}$$

Estimation of $\bar{x}_{M-i}$ requires the assumption of correct detections of all transmit symbols from x_{M-i+1} to x_M. The subchannel with the highest SNR introduces the largest interference on the remaining subchannels, and furthermore, $R_{M,M}$ has the least degree of freedom. Therefore, the Mth subchannel has the worst statistics, which limits the performance of the V-BLAST scheme and the diversity order of the system performance.

In its direct implementation, the algorithm has a complexity of order $O(M^4)$. In order to reduce the complexity by reducing "inverting" and "squaring" calculation, a "square-root" algorithm in [12] is proposed to increase the robustness and reduce the complexity to $O(M^3)$. In the "square-root" algorithm, an augmented channel matrix is established for QR decomposition:

$$\begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_M \end{bmatrix} = \mathbf{QR} = \begin{bmatrix} \mathbf{Q}_\alpha \\ \mathbf{Q}_2 \end{bmatrix} \mathbf{R} \tag{2.32}$$

where $\mathbf{Q}$ is an $(N+M) \times M$ matrix with orthonormal columns and $\mathbf{R}$ is an $M \times M$ nonsingular matrix. We have

$$\mathbf{\Sigma}^{\frac{1}{2}} = \mathbf{R}^{-1}$$

and

$$\mathbf{H}_\omega^\dagger = \mathbf{\Sigma}^{\frac{1}{2}} \mathbf{Q}_\alpha^*$$

where $\mathbf{\Sigma}^{\frac{1}{2}}(\mathbf{\Sigma}^{\frac{1}{2}})^* = \mathbf{\Sigma}$.

$\mathbf{\Sigma}^{\frac{1}{2}}$ and $\mathbf{Q}_\alpha$ can be generated by a recursion algorithm initialized with $\mathbf{\Sigma}_{|0}^{\frac{1}{2}} = \frac{1}{\sigma_n}\mathbf{I}_M$ and $\mathbf{Q}_0 = \mathbf{0}_{N\times M}$ [12], where $\mathbf{\Sigma}_{|i}^{\frac{1}{2}}$ denotes the ith round to get the matrix of $\mathbf{P}^{1/2}$. The recursion algorithm can be express as:

$$\begin{bmatrix} 1 & \mathbf{H}_i \mathbf{\Sigma}_{|i-1}^{\frac{1}{2}} \\ 0 & \mathbf{\Sigma}_{|i-1}^{\frac{1}{2}} \\ -e_i & \mathbf{Q}_{i-1} \end{bmatrix} \mathbf{\Theta}_i = \begin{bmatrix} r_{e,i}^{\frac{1}{2}} & 0 \\ \mathbf{K}_{\Sigma,i} & \mathbf{\Sigma}_{|i}^{\frac{1}{2}} \\ \mathbf{A}_i & \mathbf{Q}_i \end{bmatrix}$$

where $\mathbf{e}_i$ is the ith unit vector of dimension N (an $N \times 1$ vector of all zeros except for the ith entry, which is unity), and $\mathbf{\Theta}_i$ is any unitary transformation that transforms the first row of the pre-array to lie along the direction of the first unit row vector. $r_{e,i}^{\frac{1}{2}}$, $\mathbf{K}_{\Sigma,i}$ and $\mathbf{A}_i$ are the redundant parts after the recursion. After N steps the algorithm yields the desired results by:

$$\mathbf{\Sigma}^{\frac{1}{2}} = \mathbf{\Sigma}^{\frac{1}{2}}_{|N}$$

and

$$\mathbf{Q}_\alpha = \mathbf{Q}_N.$$

With the generation of $\mathbf{\Sigma}^{\frac{1}{2}}$ and $\mathbf{Q}_\alpha$, reorder the entries of $\mathbf{x}$ so the Mth diagonal entry of $\mathbf{\Sigma}$ is the smallest. Denote a unitary transformation $\mathbf{\Gamma}$ which rotates the Mth row of $\mathbf{\Sigma}^{\frac{1}{2}}$ to lie along the direction of the M-th unit vector, we have

$$\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Sigma}^{\frac{(M-1)}{2}} & \mathbf{\Sigma}_M^{\frac{M-1}{2}} \\ 0 & \mathbf{\Sigma}_M^{\frac{1}{2}} \end{bmatrix}$$

where $\mathbf{\Sigma}_M^{\frac{1}{2}}$ is a scalar. Denotes $\mathbf{\Sigma}^{\frac{(M-1)}{2}}$ as the square-root of $\mathbf{\Sigma}^{M-1}$. Repeat the above procedures until $\mathbf{\Sigma}^{\frac{1}{2}}$ is transferred to an upper triangular matrix. Let $\mathbf{q}_{\alpha,i}$, $i = 1, \ldots, M$ denotes the columns of $\mathbf{Q}_\alpha$, so

$$\mathbf{Q}_\alpha = [\mathbf{q}_{\alpha,1}, \ldots, \mathbf{q}_{\alpha,M}] \,.$$

The nulling vectors for the signal x_1 to x_M are given by

$$H^\dagger_{\alpha,i} = \Sigma_i^{\frac{1}{2}}\mathbf{q}^*_{\alpha,i}$$

where $\Sigma_i^{\frac{1}{2}}$ denotes the ith diagonal entry of $\mathbf{\Sigma}^{\frac{1}{2}}$.

From the procedures above, when $\mathbf{\Sigma}^{1/2}$ and $\mathbf{Q}_\alpha$ are computed, $\mathbf{H}^\dagger_\alpha$ is also computed. There is no need to recompute $\mathbf{\Sigma}$ and deflate channel matrix $\mathbf{H}^{(M-1)}$ to get the estimation of $\mathbf{x}$. This method saved a lot in the computational complexity.

CHAPTER 3. THE PROPOSED ALGORITHM

3.1 Motivation

Maximum-likelihood detection can achieve the optimal detection results but is complexity inefficient without symbol detection under nulling-canceling algorithm. On the other hand, nulling-canceling algorithm has low complexity and suboptimal performance. A combination of these two algorithms may achieve an improved performance result on both the reliability and complexity efficiency.

Reference [13] has proposed an algorithm that detects the worst several subchannels with block maximum-likelihood decoding, and then proposes the nulling-canceling detection for the remaining subchannels. This algorithm can achieve a better BER performance, but ignores the fact that the block ML detection may have a similar BER performance as nulling-canceling algorithm, especially when SNR is low. The change of the symbol estimation orders with the QR decomposition nulling-canceling algorithm may achieve independent estimation result. However, the result of the two orderings should be related intrinsically since the two estimations of different orderings all come with the same statistics. Therefore, a comparison between symbol estimates with different orderings can be used before the block maximum-likelihood detection to lower the computational complexity of the BLAST system.

3.2 The Proposed Detection Algorithm

Based on the nulling and canceling algorithm, we propose in the following a new detection algorithm that combines the original nulling-canceling detection with ML detection together. The goal is to improve the error performance upon the nulling-canceling algorithm, with small increase in the system complexity.

Let α and β be two integers larger than 1, and $\beta \leq \alpha$. Our method will first detect α of the M symbols using nulling and canceling scheme twice, with different orderings. It then compares the estimation results for these α symbols and see if they agree. If they do, then the effects of these symbols are removed from the received signal, assuming that the estimates are correct. Otherwise, maximum-likelihood detection is applied on β of the α symbols, to make a supposedly better detection of the β symbols, after first canceling out the interference from the remaining $M-\beta$ symbols. We use two parameters α and β to control how often ML detection is used and how many symbols it is used on, and do not always choose $\beta = \alpha$, for added flexibility.

Using $\alpha = \beta = 2$ as an example, we will describe the algorithm in more detail in the following.

As in the original nulling and canceling algorithm, we order the symbols according to their SNR, where the subchannel with the worst statistics has the largest SNR. This results in a QR decomposition of a column-permuted $\mathbf{H}$:

$$\mathbf{HP} = \mathbf{QR} \tag{3.1}$$

where $\mathbf{P}$ is an $M \times M$ permutation matrix with a single one on each and every row and column, $\mathbf{P}$ is decided by the SNR ordering of the transmit signal covariance matrix $\mathbf{\Sigma}$. $\mathbf{Q}$ is a $N \times M$ matrix and $\mathbf{Q}^*\mathbf{Q} = \mathbf{I}_M$, and $\mathbf{R}$ is an $M \times M$ upper triangular matrix with positive diagonals in non-decreasing order.

We use x_i to denote the ith entry of vector $\mathbf{x}$, and $R_{i,j}$ to denote the (i,j)th entry of matrix $\mathbf{R}$. Define $\bar{\mathbf{H}} = \mathbf{HP}$ and $\check{\mathbf{x}} = \mathbf{P}^T x$. In our proposed method, we estimate the last two entries of $\check{\mathbf{x}}$, namely $\check{x}_M$ and $\check{x}_{M-1}$, in two ways. In the first way, they are estimated just as in the original nulling and canceling algorithm. In the second way, $\check{x}_{M-1}$ is detected before $\check{x}_M$, by nulling out the interference from $\check{x}_1, \check{x}_2, \ldots, \check{x}_{M-2}$ and $\check{x}_M$ first. Then $\check{x}_M$ is detected after the interference from $\check{x}_{M-1}$ is removed. The last two columns of the permuted channel matrix $\bar{H}$ also need to be exchanged to get the corresponding channel matrix.

Let $[\check{x}_{M-1}^{(1)}, \check{x}_M^{(1)}]^T$ and $[\check{x}_{M-1}^{(2)}, \check{x}_M^{(2)}]^T$ denote the detected symbols using the two different orderings, respectively. We then compare to see whether $\check{x}_{M-1}^{(1)} = \check{x}_{M-1}^{(2)}$ and $\check{x}_M^{(1)} = \check{x}_M^{(2)}$. If they both agree, then we move on using either ordering to cancel out the last two symbols' interference and continue the nulling-canceling algorithm. Otherwise, a block ML detection is applied on the last two symbols by minimizing

$$\left\| \begin{bmatrix} \bar{y}_{M-1} \\ \bar{y}_M \end{bmatrix} - \begin{bmatrix} R_{M-1,M-1} & R_{M-1,M} \\ 0 & R_{M,M} \end{bmatrix} \begin{bmatrix} \check{x}_{M-1} \\ \check{x}_M \end{bmatrix} \right\| \tag{3.2}$$

where $\bar{y}_{M-1} = q^*_{M-1}y$, $\bar{y}_M = q^*_M y$, and q_{M-1} and q_M are the $(M-1)$st and Mth columns of $\mathbf{Q}$.

Note that in the second ordering scheme, where $\check{x}_{M-1}$ is detected first instead of $\check{x}_M$, we do not need to perform a new QR decomposition of the permutation of the matrix $\bar{\mathbf{H}}$. Instead, we only need to upper-triangularize the column-switched 2×2 matrix

$$\mathbf{R}' = \begin{bmatrix} R_{M-1,M}, & R_{M-1,M-1} \\ R_{M,M}, & 0 \end{bmatrix} \tag{3.3}$$

by applying a Householder transformation to the first column of it. The same Householder transformation should also be applied to $[\bar{y}_{M-1}, \bar{y}_M]^T$.

The Householder transformation can be used to obtain a QR decomposition or to bring a matrix to an upper-triangularize matrix by reflecting the first one column of a matrix onto a multiple of a standard basis vector, reducing some entries of the vector to zero and keeping the norm of the vector unchanged. Denotes the first row of $\mathbf{\Sigma}^{\frac{1}{2}}$ as $\mathbf{u} = \mathbf{\Sigma}_1^{\frac{1}{2}}$. u_1 is the first entry of $\mathbf{u}$. Recalculate u_1 to get the householder reflector $\mathbf{u}$, where

$$\hat{u}_1 = u_1 + sgn(u_1)(\| \Sigma_1^{\frac{1}{2}} \|_2)^{\frac{1}{2}}.$$

Function $\mathrm{sgn}(x)$ is the sign function.

We use $\hat{u}_1$ to substitute u_1 in $\mathbf{u}$. The normalized $\mathbf{u}$, householder reflector $\hat{u}$ can be calculated by

$$\hat{\mathbf{v}} = \frac{\mathbf{u}}{\| \mathbf{u} \|}.$$

Therefore, the householder matrix $\mathbf{Z}$ can be calculated by

$$\mathbf{Z} = \mathbf{I}_2 - 2\hat{\mathbf{v}}\hat{\mathbf{v}}^*. \tag{3.4}$$

Applying the householder transformation to the first column of $\mathbf{R}'$ in (3.3), we have

$$\mathbf{Z}\begin{bmatrix} R_{M-1,M} \\ R_{M,M} \end{bmatrix} = \begin{bmatrix} l_1 \\ 0 \end{bmatrix} \tag{3.5}$$

where

$$l_1 = \left\| \begin{pmatrix} R_{M-1,M} \\ R_{M,M} \end{pmatrix} \right\|_2 \tag{3.6}$$

The $\mathbf{R}$ matrix for the QR decomposition of the second ordering scheme is

$$\mathbf{R}^{''} = \mathbf{Z}\mathbf{R}^{'} = \begin{bmatrix} l_1 & v_1 \\ 0 & v_2 \end{bmatrix} \tag{3.7}$$

where

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{Z}\begin{bmatrix} R_{M-1,M-1} \\ 0 \end{bmatrix} \tag{3.8}$$

Also, denote $\bar{\mathbf{y}} = [\bar{y}_{M-1}, \bar{y}_M]^T$, the resorted $\bar{\mathbf{y}}$ in the second ordering can be represented as:

$$\bar{\mathbf{y}}^{'} = \mathbf{Z}\bar{\mathbf{y}} \tag{3.9}$$

When $\alpha = \beta = 3$, let $[\check{x}^{(1)}_{M-2}, \check{x}^{(1)}_{M-1}, \check{x}^{(1)}_{M}]^T$ and $[\check{x}^{(2)}_{M-2}, \check{x}^{(2)}_{M-1}, \check{x}^{(2)}_{M}]^T$ denote the detected symbols using the two different orderings, respectively. We then compare to see whether $\check{x}^{(1)}_{M-2} = \check{x}^{(2)}_{M-2}$, $\check{x}^{(1)}_{M-1} = \check{x}^{(2)}_{M-1}$ and $\check{x}^{(1)}_{M} = \check{x}^{(2)}_{M}$. If they both agree, we cancel out the last three symbols' interference and continue the nulling-canceling algorithm for the rest columns. Otherwise, a block ML detection is applied on the last three symbols.

A similar description can also be given for a system with $\alpha = \beta > 3$.

When $\beta < \alpha$, we will do the detection of the first α symbols in two ways, using two different orderings, but ML detection is only applied to the last β symbols if the comparison of the detection results do not agree. Original nulling-canceling algorithm will be applied on the remaining $M - \alpha$ subchannels (if comparison results agree with each other) or the remaining $M - \beta$ subchannels (if comparison results do not agree and block ML detection is applied). Compared with the $\beta = \alpha$ scheme where α has the same value, this scheme can reduce the complexity but the BER performance also becomes worse since less subchannels are detected by block ML detection on average.

The algorithm can be summarized as follows:

1. Perform the ordered QR decomposition of $\mathbf{H}$ as in (3.1), using the square-root algorithm [12] (see Chapter 2.3.2 for detail).

2. Set $\bar{\mathbf{y}} = \mathbf{Q}^*\mathbf{y}$.

3. For i from 0 to $\alpha - 1$, decide $\check{x}^{(1)}_{M-i}$ based on

$$\bar{y}_{M-i} - \sum_{j=0}^{i-1} R_{M-i,M-j}\hat{x}^{(1)}_{M-j} = R_{M-i,M-i}\check{x}_{M-i}.$$

4. Perform the QR decomposition of the lower right $\alpha \times \alpha$ submatrix of $\mathbf{R}$ with reversed columns.

5. Detect the last α symbols again, using the new ordering to obtain $\check{x}^{(2)}_{M-i}$, $i = \alpha - 1, \alpha - 2, \ldots, 0$.

6. Compare $\check{x}^{(1)}_{M-i}$ with $\check{x}^{(2)}_{M-i}$, $i = 0, 1, \ldots, \alpha$. If they all agree, continue the detection of the remaining $M - \alpha$ symbols. If they do not agree, perform block ML detection of the last β symbols by minimizing a second norm similar to (3.2). Continue with nulling and canceling on the remaining $M - \beta$ symbols.

We remark that the square-root algorithm was derived such that $\mathbf{R}^{-1}$ is obtained in the permuted and ordered QR decomposition. The Householder transformation that need to be applied to (3.3) can be transformed to be performed on the lower-right sub-block of R^{-1} instead, and yield a modified sub-block for the locally reversely ordered symbols.

3.3 Variation

For $\alpha = 2$ case, since the last two entries of $\bar{\mathbf{x}}$ are chosen, the second ordering scheme is unique with no ambiguity. However, for α equals or more than 3, the ordering in which the subchannels are decoded and canceled is not unique. Though the performance improvement by different orderings may not be as much as that by combined block ML detection, it may still appear. Therefore, discussion of the ordering is important and shall be beneficial to this thesis.

Given $\alpha = 3$ and $\beta = 3$, despite the regular detection of $\bar{\mathbf{x}}$ which is from $\check{x}_M$ to $\check{x}_1$, the second ordering scheme detects $\check{x}_{M-2}$ first, by nulling out the interference from $\check{x}_1, \check{x}_2, \ldots, \check{x}_{M-3}$ and $\check{x}_{M-1}, \check{x}_M$. $\check{x}_{M-1}$ is detected after the interference from $\check{x}_{M-2}$ is removed. And then, $\bar{x}_M$ is detected after $\check{x}_{M-1}$ is removed.

To compare the performance of different detection orderings, we detect the $\check{x}$ under the regular detection ordering from $\check{x}_M$ to $\check{x}_1$ first. The comparison group detects $\check{x}_{M-1}$ first, by nulling out the interference from $\check{x}_1, \check{x}_2, \ldots, \check{x}_{M-2}$ and $\check{x}_M$. Then, $\check{x}_{M-2}$ is detected after the interference from $\check{x}_{M-1}$ is removed. And finally, $\check{x}_M$ is detected after $\check{x}_{M-2}$ is removed.

The BER performance is always limited by the first detected subchannel. The first detected symbol $\check{x}_{M-1}$ in the detection ordering of $\check{x}_{M-1}$, $\check{x}_{M-2}$ and then $\check{x}_M$ has a better SNR compares with the first detected symbol in the order of $\check{x}_{M-2}$, $\check{x}_{M-1}$ and then $\check{x}_M$. Therefore, nulling-canceling with the first group requires less time in combined block ML detection. It should also yield a worse BER performance (since combined block ML detection is optimal than nulling-canceling algorithm).

A similar analysis can also be extended to $\alpha > 3$ and $\beta > 3$, with the same consideration of the first detected subchannel. The BER performance results is also foreseeable.

We also perform another detection algorithm, which compares the estimation results between the unordered and the ordered detection symbols. For $\alpha = 2$, assume the last two entries of $\check{\mathbf{x}}$, $\check{x}_M^{(1)}$ and $\check{x}_{M-1}^{(1)}$ correspond to the x_{i_1} and x_{i_2} in the original unordered transmit symbol matrix. $\bar{x}_{i_1}$ and $\bar{x}_{i_2}$ is the detection of x_{i_1} and x_{i_2} under QR decomposition method. Then we compare to see whether $\check{x}_M^{(1)} = \bar{x}_{i_1}$ and $\check{x}_{M-1}^{(1)} = \bar{x}_{i_2}$. If they both agree, then the interference of the last two symbols will be canceled and nulling-canceling algorithm with be applied on the rest columns. Otherwise, the block ML detection will be applied on the detection of the last two symbols, following with the nulling-canceling of the rest symbols. Also consider the detected symbols of $\check{x}_{M-1}^{(2)}$ and $\check{x}_M^{(2)}$, which is detected by QR decomposition with different ordering as presented above. Compare to see whether $\check{x}_M^{(2)} = \bar{x}_{i_1}$ and $\check{x}_{M-1}^{(2)} = \bar{x}_{i_2}$. Follow the same way, either interference of the last two symbols will be canceled and nulling-canceling algorithm applied, or block ML detection will be applied on the last two columns, follow with the nulling-canceling of the rest symbols.

CHAPTER 4. PERFORMANCE ANALYSIS

Among all the MIMO system signal processing algorithms, V-BLAST is an important one due to its good complexity-performance tradeoff. In this section, we perform some analysis of the computational complexity and performance of our proposed algorithm.

4.1 Complexity Analysis

For computational complexity computation, only the multiplications are considered in the complexity computation. For each complex multiplication, we count the total number of multiplications as four [25].

The complexity of our proposed algorithm can be considered as four parts:

1. Determine the nulling vectors and optimal ordering

The square-root algorithm proposed in [12] has a computational complexity of: $\frac{2}{3}M^3 + 7NM^2 + 2N^2M$. When $M = N$, the complexity reduces to $\frac{29}{3}M^3$.

2. Left multiply receive signal by $\mathbf{Q}^*$

The left multiplication of received signal $\mathbf{y}$ by $\mathbf{Q}^*$ has a complexity of $2MN$ in the total. When $M = N$, the complexity reduces to $2M^2$.

3. Selective comparison

Calculation of $\bar{x}_i$ requires a complexity of $M + 1 - i$. The total complexity in the selective comparison part is $2\Sigma_{m=1}^{\alpha-1} m = \alpha(\alpha - 1)$, which is small and can be neglected.

4. Block ML estimation

For L-QAM system, the block ML selection requires approximately a complexity in the order of βL^β [14].

In practice, the decoding algorithm is usually performed on a number of blocks. For some of these blocks, block ML detection of β out of the M symbols will be performed. Such block ML detection needs to be initiated if the two nulling-canceling algorithms with different orders cannot agree on the first few α detected symbols. Since block ML detection is a costly operation, we would like to have as few ML detection blocks as possible from the complexity point of view. From the performance point of view, we would like to have as many as possible.

In some cases, when system reliability is much more important than the complexity (battle field, satellite communication etc.), the number of block ML detection β can be higher than the number of nulling-canceling detection and comparison parameter α to ensure reliable transmission. This scheme indicates that we may include some symbols for block ML detection that were not considered in the nulling-canceling and comparison.

To measure the percentage of ML detection blocks, we can introduce an empirical parameter $\epsilon = N^{(\mathrm{ML})}/N^{(\mathrm{total})}$, where $N^{(\mathrm{ML})}$ is the number of blocks for which an ML detection was initiated, and $N^{(\mathrm{total})}$ denotes the total number of blocks decoded. The value of ϵ will be empirically determined in simulation to give an indication of the complexity of the algorithm, and can be controlled by choosing α and β. When $M = N$, the computational complexity per block of our proposed decoding algorithm is in the order of $\mathcal{O}(M^3) + \epsilon\beta L^\beta$, or more precisely, with the expression

$$\frac{29}{3}M^3 + 2M^2 + \epsilon\beta L^\beta. \tag{4.1}$$

Denote Δ as the complexity of block ML detection over the total complexity, where

$$\Delta = \frac{\epsilon\beta L^\beta}{\frac{29}{3}M^3 + 2M^2 + \epsilon\beta L^\beta} \times 100\% \tag{4.2}$$

Table 4.1 Complexity of block ML detection over the total complexity in proposed algorithm ($M = N$)

M	β	L	ϵ	Δ (%)
4	2	4	0.1	1.13
8	2	16	0.1	1.00
4	3	16	0.01	15.89
8	3	16	0.01	2.36

Table 4.1 shows the complexity of block ML detection over the total complexity under our proposed algorithm and different parameters. When constellation becomes complex, lowering the ϵ threshold can still reduce the complexity allocated on the block ML calculation.

As shown in [13], if we always perform ML detection on the last β symbols, we should expect to obtain a diversity order β. But here, ML detection is only performed when we could not reach an agreement on the last few symbols by using low complexity nulling and canceling algorithm. As a result, the diversity order of our proposed algorithm is usually smaller than β.

4.2 Performance Analysis

In this section, we perform the theoretical BER analysis for the nulling-canceling and maximum-likelihood detections respectively. A completed BER analysis is given by [13] under zero-forcing (same as the QR decomposition of channel matrix H model in our thesis) and maximum-likelihood detection, from which the performance of the nulling-canceling model is always limited by the worst subchannel and performing block ML for β symbols can increase the diversity order to β. However, how ordering affects the BER in nulling-canceling scheme does not have a clear explanation, only the upper bounds of the BERs were derived by [13] respectively. In the last part of this section, we present a theoretical BER for an unordered hybrid algorithm with nulling-canceling of α subchannels and block ML detection of β subchannels under BPSK and 4-QAM constellations respectively.

4.2.1 BER analysis for ordered system

4.2.1.1 BER analysis for nulling-cancelling

Denotes E_i as the error event in the ith subchannel, $P\left[E_i\right]$ as the probability of E_i. $\mathbf{R}$ is the $M \times M$ matrix after QR decomposition, where $R_{M,M}$ corresponds to the worst channel condition, with the error event E_M. $x_i^{[1]}$ is the symbol corresponding to the ith subchannel,

$x_i^{[2]}$ is a nearest neighbor of $x_i^{[1]}$. From the result of [13], for large SNR and $M = N$, we have

$$P[E_i] \leq P[E_M] \times \frac{1}{1-\zeta}, \quad \text{for} \quad \forall i \tag{4.3}$$

where

$$\zeta = \frac{1}{1 + d_{\min}^2/4\sigma_n^2}$$

is a small positive number when SNR is large, in which d_{min} is considered as the distance between nearest neighbors which is the same for all the subchannels.

Equation (4.3) shows that the performance of the nulling-canceling algorithm is always limited by the worst subchannel, namely, the channel M after ordering.

4.2.1.2 BER analysis for ML detection

Denotes E_β as the error event for the β symbols $(x_{M-\beta+1}, \ldots, x_M)$ that are detected by combined block ML algorithm. Given $M = N$ and follow the same procedures of BER analysis for nulling-cancelling algorithm, [13] shows that

$$P[E_i] \leq P[E_\beta] + \delta, \quad \text{for} \quad i = 1, 2, \ldots, n - \beta$$

where δ is a small positive number.

The error probability $P[E_\beta]$ is upper bounded by

$$\left[\frac{L}{1 + d_{\min}^2/4\sigma_n^2}\right]^\beta$$

where L is the constellation size and β is the subchannel number that we chose to perform the combined block ML detection.

This result shows that combined block ML detection of β subchannels will increase the system diversity order to β. The result is for $M = N$. For $M \neq N$, the diversity order of combined block ML detection of β channels is $N - M + \beta$.

4.2.1.3 BER analysis for the proposed algorithm

It is hard to get a closed form expression for the error probability of our proposed algorithm, since ordered system and comparison between symbol estimates of different orderings does not have a closed form expression on BER performance. However, the BER performance result presented by [13] can be applied on our proposed algorithm.

Since the performance of the nulling-canceling algorithm is always limited by the worst subchannel, when $\alpha = 2$, the equation (4.3) can be modified as

$$P\left[E_i\right] \leq \min(P\left[E_M\right], P\left[E_{M-1}\right]) \times \frac{1}{1-\zeta}, \quad \text{for} \quad \forall i \tag{4.4}$$

where $P\left[E_{M-1}\right]$ is the detected symbol with the second largest post-estimation SNR.

When SNR is large enough, which means ζ is very close to zero, from the result of equation (4.3), the equation (4.4) can be simplified as

$$P\left[E_i\right] \leq P\left[E_{M-1}\right] \times \frac{1}{1-\zeta}, \quad \text{for} \quad \forall i \tag{4.5}$$

Therefore, for our proposed system with comparison of α subchannels with the SNR ordering and different ordering schemes, and block ML detection of β subchannels out of α subchannels when the comparison can not achieve agreement, the upper bound of BER performance of any detected subchannel P_{e_i} can be express as (for high SNR)

$$P_{e_i} \leq \{(1-\epsilon)P\left[E_j\right] + \epsilon P\left[E_\beta\right]\} \times \frac{1}{1-\zeta}, \quad \text{for} \quad \forall i \tag{4.6}$$

where j corresponds to the first detected symbol's subscript number in the second ordering scheme for an SNR descending ordered signal vector under nulling-canceling algorithm, $j \leq M$.

4.2.2 BER analysis for unordered system

Since how ordering may affect the BER may not have a clear explanation, in this section, we focus on the BER analysis of a joint detection algorithm in a system with unordered transmit signal vector, where a combined outage probability expression $P_{e,total}$ is generated with the joint consideration of nulling-canceling α unordered subchannels and combined block ML detection of β unordered subchannels. BPSK constellation and Rayleigh fading channel are assumed in the derivation process. Error probability for a 4-QAM (QPSK) constellation Rayleigh fading channel will be given below.

4.2.2.1 BER analysis for BPSK fading channel

For an unordered system with nulling-canceling algorithm, a BER performance analysis is presented in [26] and [27] with the ascending detection (from x_1 to x_M) of the transmit symbols. It is the same way to derive the closed form expression for the descending detection (from x_M to x_1) of the transmit symbols.

Assume we detect x_M first, denote:

$$R_{i,\{M-k+1,M-k+2,\ldots,M\}} \doteq \{i \text{ errors occurred in}$$
$$\text{detecting subchannels } M-k+1, M-k+2, \ldots, M\}.$$

The exact error probability of the kth detected subchannel with BPSK modulation and Rayleigh fading is [18]

$$P_e(D,\gamma) = \left[\frac{1}{2}(1-\sqrt{\frac{\gamma}{1+\gamma}})\right]^D \sum_{t=0}^{D} \begin{pmatrix} D-1+t \\ t \end{pmatrix} \left[\frac{1}{2}(1+\sqrt{\frac{\gamma}{1+\gamma}})\right]^t. \tag{4.7}$$

where $D = N - M + k$ is the diversity order and $\gamma = \frac{E_s}{N_0+4iE_s}$. E_s is the transmit energy per bit.

Assume $k \leq \alpha$, denote $P_{e,k}$ as the error probability of the kth detected subchannel under the nulling-canceling algorithm. $P_{e,k}$ can be calculated from a recursive derivation beginning from

$$\Pr(R_{0,\{M\}}) = 1 - P_e(N-M+1, \frac{E_s}{N_0}) \tag{4.8}$$

and

$$\Pr(R_{1,\{M\}}) = P_e(N-M+1, \frac{E_s}{N_0}) \tag{4.9}$$

which are similar in Table 3.1 of [27]. The expression of $P_{e,k}$ is:

$$P_{e,k} = \sum_{i=0}^{K-1} P_e(N-M+k, \frac{E_s}{N_0+4iE_s}) \Pr(R_{i,\{M-k+1,M-k+2,\ldots,M\}}) \tag{4.10}$$

For the unordered system with combined block ML detection, where β channels are detected simultaneously. The diversity order or the kth detected channel (where $k \leq \beta$) is always β.

Therefore, the error probability of the kth detected channel after block ML detection of β channels is

$$P_{e,ML} = \sum_{i=0}^{k-1} P_e(\beta, \frac{E_s}{N_0 + 4iE_s}) \Pr(R_{i,\{M-k+1,M-k+2,\dots,M\}}) \tag{4.11}$$

Recall the block ML detection parameter ϵ defined in Chapter 4.1, which indicates the time allocated in block ML detection. Define $P_{e,total}$ as the total error probability of the kth detected subchannel with nulling-canceling of α subchannels and block ML detect of β subchannels (all based on descending SNR-ordering and ZF filtering), the total error probability for an unordered system with BPSK constellation and Rayleigh fading channel is

$$P_{e,total} = (1-\epsilon)P_{e,k} + \epsilon P_{e,ML} \tag{4.12}$$

4.2.2.2 BER analysis for 4-QAM fading channel

A BER performance of M-PSK constellation and Rayleigh fading channel is generalized in [18]. A 4-QAM constellation can be considered as a 4-PSK modulation.

With the detection from the Mth subchannel to the 1st subchannel, the exact error probability of the kth detected subchannel with 4-QAM modulation and Rayleigh fading is [18]

$$P_e(D,\mu) = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} \sum_{t=0}^{D-1} \begin{pmatrix} 2t \\ t \end{pmatrix} \left(\frac{1-\mu^2}{4-2\mu^2}\right)^t\right] \tag{4.13}$$

Replace $D = N - M + k$ and $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$, where $\bar{\gamma}$ is the average received SNR per channel, varies depends on different channel characteristics, equation (4.13) can be rewritten as

$$P_e(D,\bar{\gamma}) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}} \sum_{t=0}^{D-1} \begin{pmatrix} 2t \\ t \end{pmatrix} \left(\frac{1}{4+2\bar{\gamma}}\right)^t\right] \tag{4.14}$$

Consider the detection of symbol Re($\sqrt{\frac{E_s}{2}}$)+Im($\sqrt{\frac{E_s}{2}}$) in a 4-QAM constellation, for simplicity, assume $\Pr(x_j - \hat{x}_j = 0) = 1 - P_{e,k}$, $\Pr(x_j - \hat{x}_j = \mathrm{Re}(\sqrt{2E_s})) = P_{e,k}/2$ and $\Pr(x_j - \hat{x}_j = \mathrm{Im}(\sqrt{2E_s})) = P_{e,k}/2$ (the nearest two symbols -Re($\sqrt{\frac{E_s}{2}}$)+Im($\sqrt{\frac{E_s}{2}}$) and Re($\sqrt{\frac{E_s}{2}}$)-Im($\sqrt{\frac{E_s}{2}}$) comes with the same probability in the detection, and the error probability of getting the furthest symbol -Re($\sqrt{\frac{E_s}{2}}$)-Im($\sqrt{\frac{E_s}{2}}$) can be ignored).

Follow the similar derivation from [27] and above assumption, $P_{e,k}$ can be calculated from the above derivations:

$$P_{e,k} = \sum_{i=0}^{K-1} P_e(N - M + k, \frac{E_s}{N_0 + 2iE_s}) \Pr(R_{i,\{M-k+1,M-k+2,\ldots,M\}}) \tag{4.15}$$

with the initial value of

$$\Pr(R_{0,\{M\}}) = 1 - P_e(N - M + 1, \frac{E_s}{N_0}) \tag{4.16}$$

$$\Pr(R_{1,\{M\}}) = P_e(N - M + 1, \frac{E_s}{N_0}). \tag{4.17}$$

For the unordered system with block ML detection and M-PSK modulation, an average BER with multichannel reception scheme in [28] can be used for error performance calculation. Assume the detected β subchannels are statistically independent, the average BER of the block ML detection can be calculated by a β-fold integration over the joint pdf of the instantaneous SNR sequence of each subchannel

$$P_{e,ML} = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{\beta-fold} P_b(\{\gamma_l\}_{l=1}^{\beta}) \prod_{l=1}^{\beta} P_e(\gamma_l; \bar{\gamma}_l, i_l) d\gamma_1 d\gamma_2 \ldots d\gamma_\beta \tag{4.18}$$

where i_l represents the fading parameters associated with the lth detected channel. (4.18) can be simplified by a moment generation function (MGF) expression

$$P_{e,ML} = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^{\beta} M_{i_l}(-\frac{g}{\sin^2\phi}; \bar{\gamma}_l) d\phi \tag{4.19}$$

where $g = 3/(2(M-1))$ for QAM constellation [29]. For 4-QAM, $g = 1/2$ and

$$M_{i_l}(-\frac{g}{\sin^2\phi}; \bar{\gamma}_l) = (1 + \frac{g\bar{\gamma}_l}{\sin^2\phi})^{-1} \tag{4.20}$$

The error probability of the kth detected subchannel with block ML detection of β subchannels can be considered as the average BER of the block ML detection of β subchannels.

Define $P_{e,total}$ as the total error probability of the kth detected channel with nulling-canceling of α subchannels and block ML detect of β subchannels (all based on descending SNR-ordering and ZF filtering) under 4-QAM constellation and Rayleigh fading channel, the total error probability for the unordered system is

$$P_{e,total} = (1 - \epsilon)P_{e,k} + \epsilon P_{e,ML} \tag{4.21}$$

CHAPTER 5. SIMULATION RESULTS AND DISCUSSIONS

In this section, we evaluate the performance of the proposed iterative detection scheme via numerical simulations. We choose $M = N = 4$ or $M = N = 8$. The constellation size is either $L = 4$ or $L = 16$. We assume that the channel gains remain constant over one block of transmission. One vector symbol consists of M QAM symbols. One block consists of 1 vector symbol, and the number of blocks transmitted is determined by the expression: number of blocks $\times$ SNR (in dB scale) ≤ 20000.

In our simulation, we give the "square-root" algorithm in [12] with the legend of "original nulling-canceling", and the algorithm in [13] with the legend of "Combined Block ML-DFE t", where t is the number of symbols chosen in the block ML detection. The legend of our proposed algorithm is given in the form of "Proposed: alpha $=\alpha$, beta$=\beta$". Generally, the complexity parameter ϵ denoted in Chapter 4 is the same for the same channel setup and same α value, regardless of the β value.

Fig. 5.1 depicts the BER performance of a 4×4 system with 16-QAM constellation and MMSE-based ordering. The curves are shown by original nulling-canceling, $t = 2$, $t = 3$, $\alpha = \beta = 2$, $\alpha = \beta = 3$ and $\alpha = 3, \beta = 2$ respectively. About 5-7 dB gain is possible with our proposed schemes compared to the original nulling-canceling algorithm. All our proposed algorithms yield a same diversity order when SNR is high. Block ML-DFE results are the lower bounds of our proposed algorithm. There is no diversity improvement between our proposed algorithms and the original nulling-canceling algorithm.

Fig. 5.2 depicts the BER performance of a 4×4 system with 4-QAM constellation and MMSE-based ordering. The curves are shown by original nulling-canceling, $t = 2$, $t = 3$, $\alpha = \beta = 2$, $\alpha = \beta = 3$ and $\alpha = 3, \beta = 2$ respectively. The performance gain between our

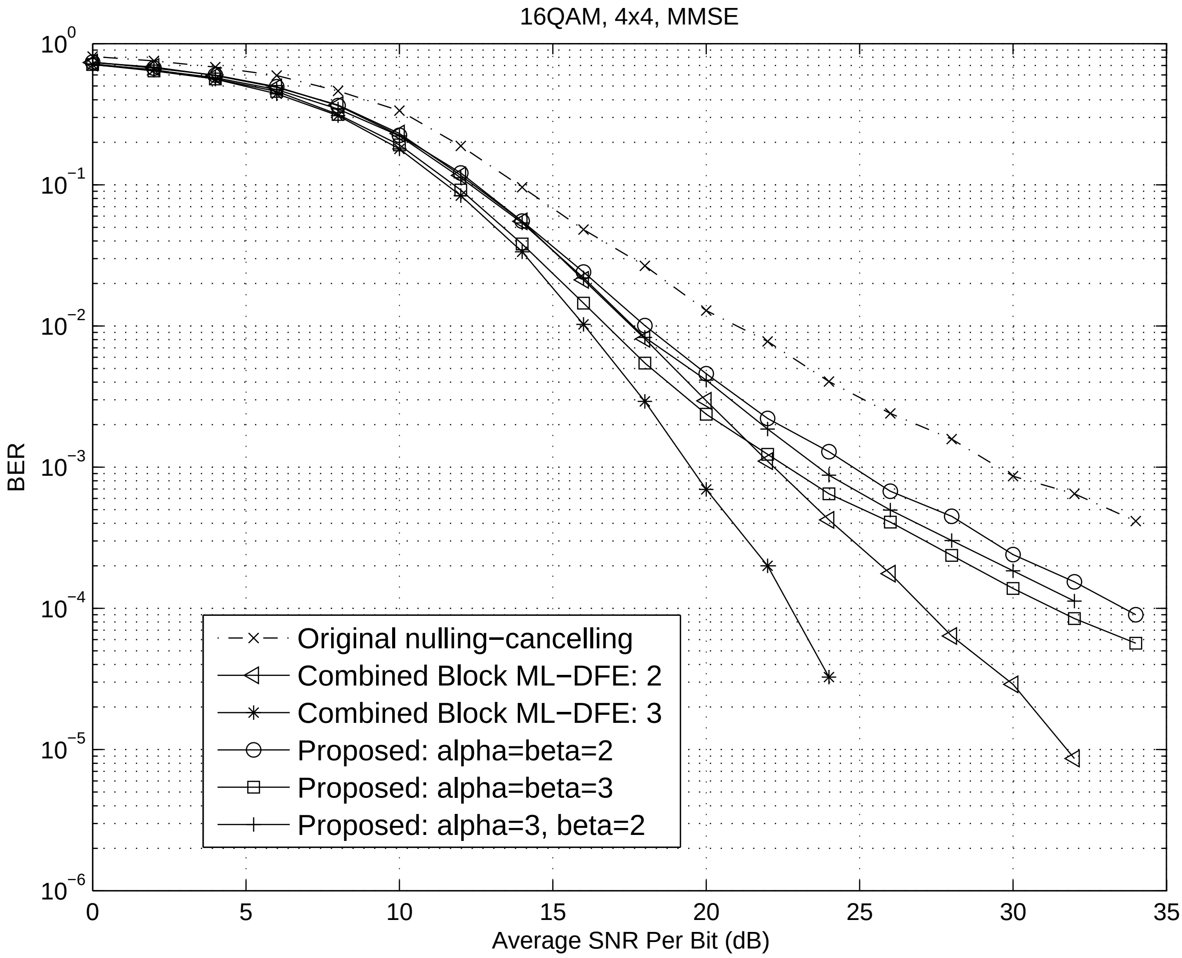

Figure 5.1 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE

proposed algorithm and original nulling-canceling algorithm is between 1-2 dB, which is less than the result in Fig. 5.1. Compare the results of $\alpha = \beta = 3$ and $\alpha = 3, \beta = 2$, we can find that with choosing the same α number, the more number of β adopted, the better BER performance is acquired.

Fig. 5.3 depicts the BER performance of a 8×8 system with 4-QAM constellation and MMSE-based ordering. The curves are shown by original nulling-canceling, $t = 2$, $t = 3$, $\alpha = \beta = 2$, $\alpha = \beta = 3$ and $\alpha = 3, \beta = 2$ respectively. The performance gain between our proposed algorithm and original nulling-canceling algorithm is less than 1 dB. The performance increase begins to be smaller because combined block ML-DFE methods provide a lower bound

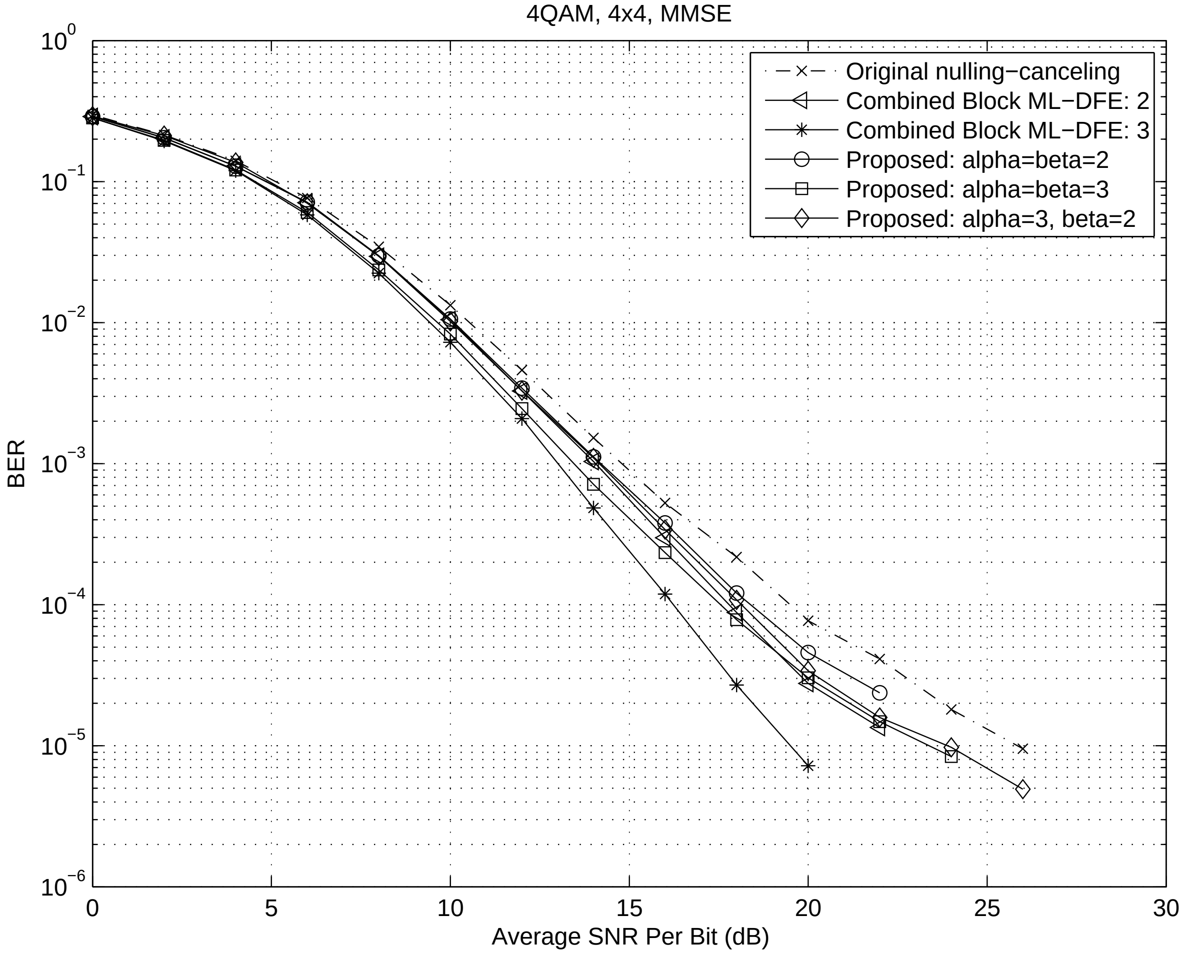

Figure 5.2 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 4-QAM, MMSE

of our proposed algorithm, which is much close to the original nulling-canceling algorithm.

Fig. 5.4 depicts the BER performance of a 4×4 system with 4-QAM constellation and ZF-based ordering. The curves are shown by original nulling-canceling, $t = 2$, $t = 3$, $\alpha = \beta = 2$, $\alpha = \beta = 3$ and $\alpha = 3, \beta = 2$ respectively. The performance gain between our proposed algorithm and original nulling-canceling algorithm is between 5 to 7 dB. The performance gain here is much bigger than the same setup but MMSE-based ordering, that is because ZF is suboptimal than MMSE in symbol detection.

Fig. 5.5 depicts the BER performance of a 4×4 system with 4-QAM constellation and MMSE- or ZF-based ordering. For both MMSE and ZF, there are three curves: one for

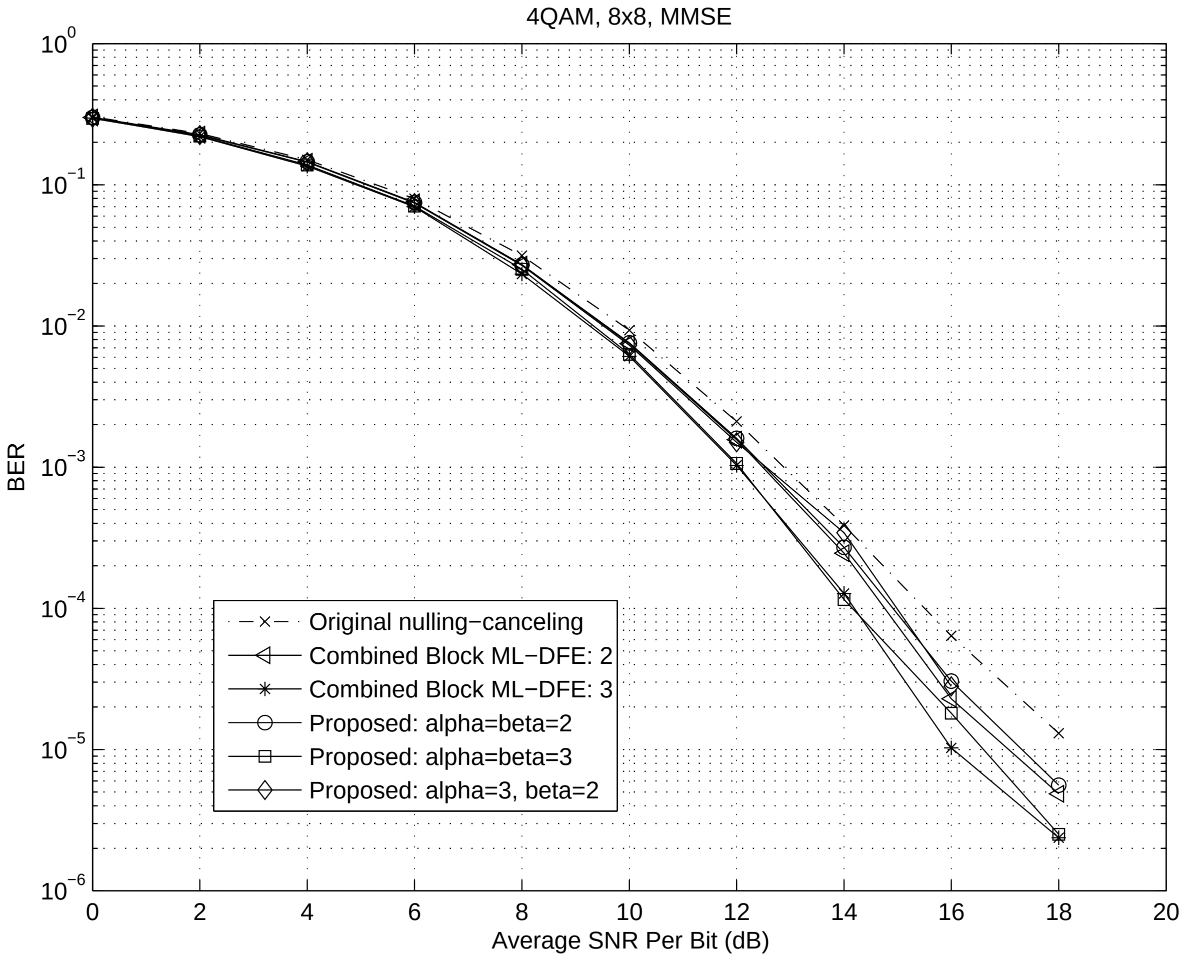

Figure 5.3 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 8$; 4-QAM, MMSE

the original nulling-canceling algorithm, one for the proposed algorithm with $\alpha = \beta = 3$, and one for the scheme in [13] where block ML detection is always performed on the first 3 symbols under SNR ordering. Compare to the original nulling-canceling algorithm, the proposed algorithm shows an improvement in performance, and the gain is higher for the ZF-based ordering scheme, which is because ZF is suboptimal than MMSE based ordering in the nulling-canceling algorithm.

Fig. 5.6 depicts the BER performance of a 4×4 system with 16-QAM constellation and MMSE-based ordering, where we detect $\alpha = 3$ symbols first by using SNR descending nulling-canceling algorithm, then compare the result with two different ordering schemes listed in

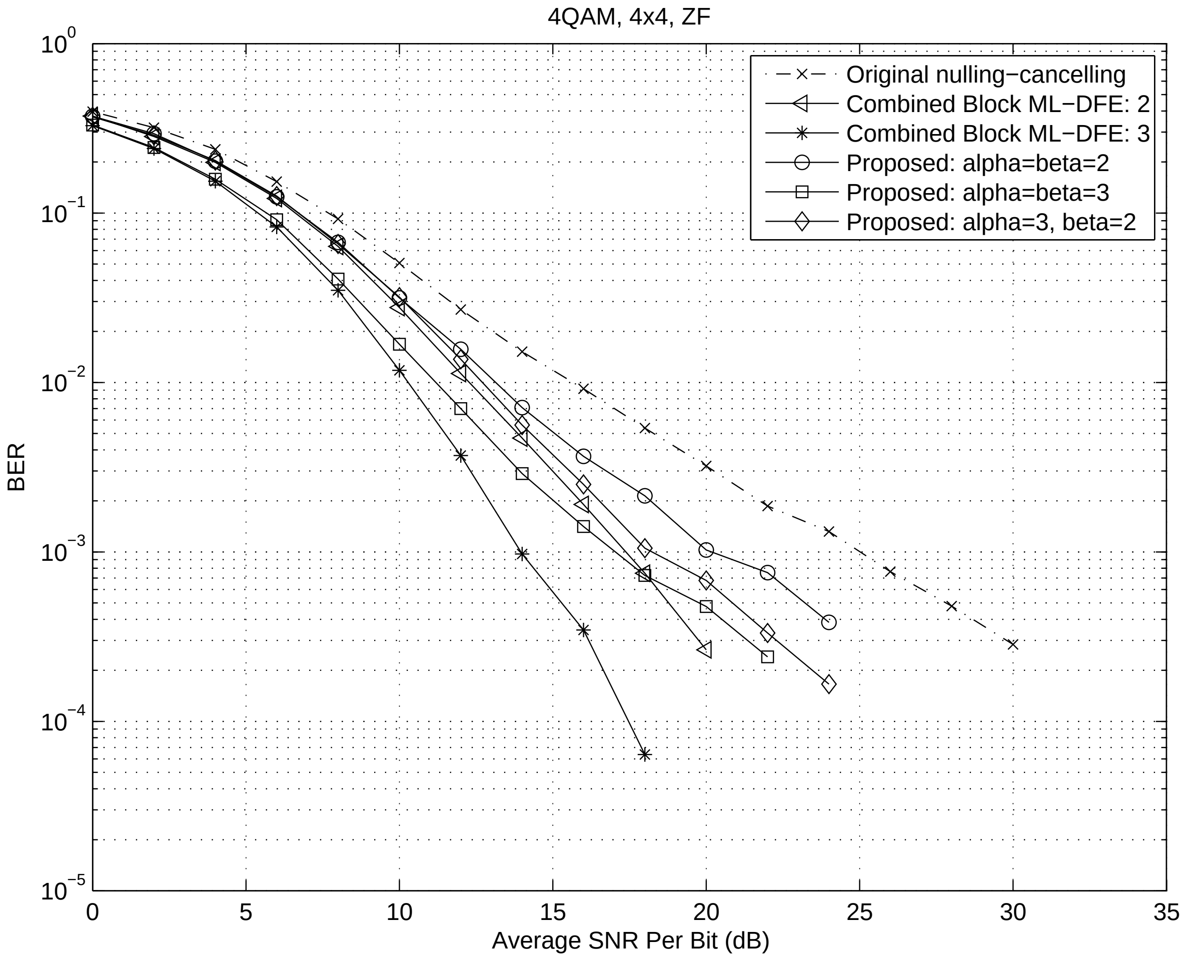

Figure 5.4 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 4-QAM, ZF

Chapter 3.3 respectively. $\beta = 2$ and $\beta = 3$ are shown together in this simulation. The solid lines show the comparison with the detection ordering of $\bar{x}_{M-2}$, $\bar{x}_{M-1}$ and $\bar{x}_M$. The dashdot lines show the comparison with the detection ordering of $\bar{x}_{M-1}$, $\bar{x}_{M-2}$ and $\bar{x}_M$. The result shows that when $\alpha = \beta = 3$, the performance with the comparison order of $\bar{x}_{M-2}$, $\bar{x}_{M-1}$ and $\bar{x}_M$ is better than the performance with the comparison order of $\bar{x}_{M-1}$, $\bar{x}_{M-2}$ and $\bar{x}_M$. This result corresponds with the analysis in the Chapter 3.3. However, for the $\alpha = 3, \beta = 2$ case, there is no significant differences between the results of different ordering schemes comparison. That is simply because the β number is smaller than the α number in the simulation. Therefore, the difference in performance caused by the ordering scheme does not all reflected on the block

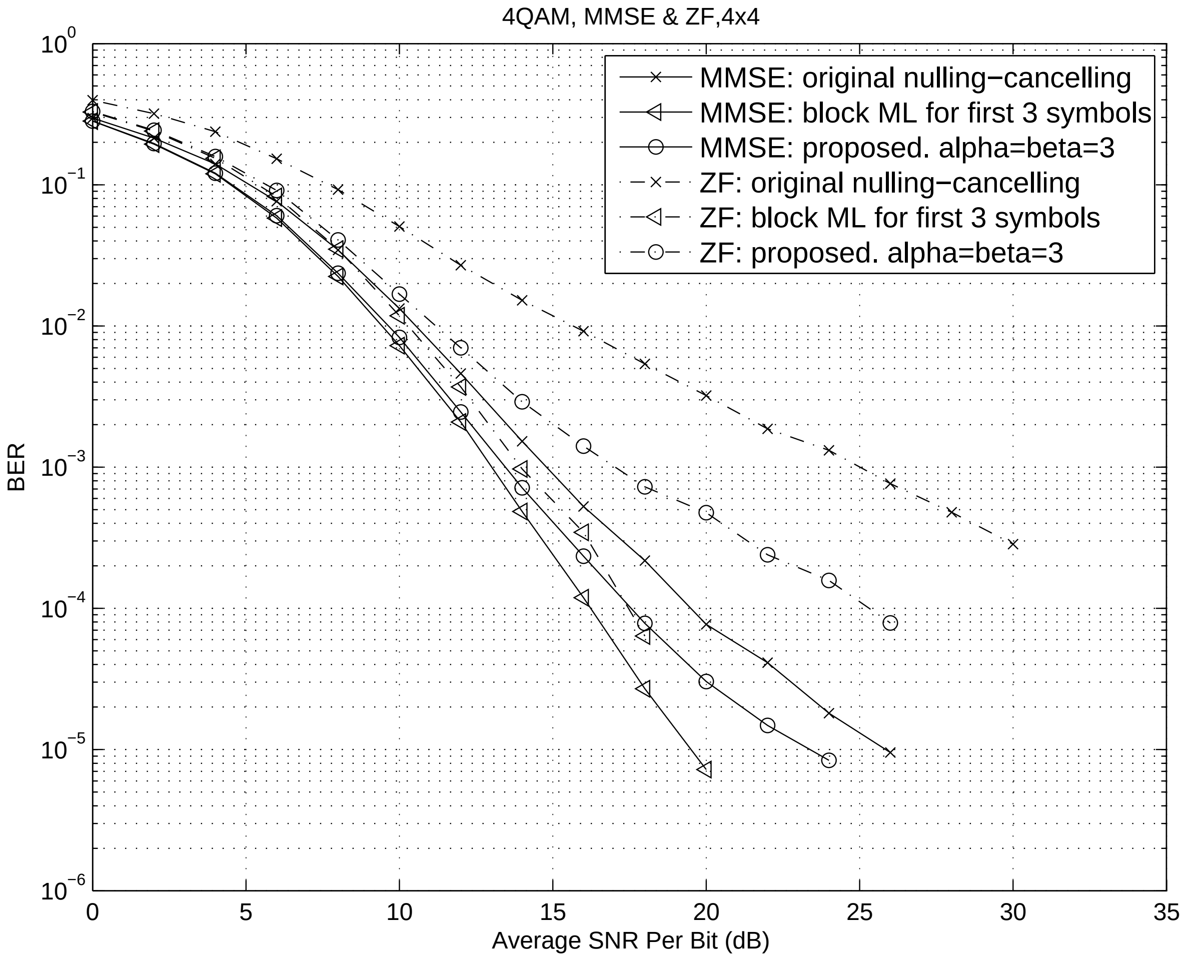

Figure 5.5 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 4-QAM, MMSE & ZF

ML step.

Fig. 5.7 depicts the BER performance of a 4×4 or 8×8 system with 4-QAM constellation and ZF-based ordering, under original nulling-canceling, $\alpha = \beta = 3$ and combined block ML-DFE=3 respectively. 4×4 system has a better performance than 8×8 system at low SNR level. However, for high SNR scenario, the more transmit and receive antennas, the better BER performance a 8×8 system may achieve.

Fig. 5.8 depicts the percentage of block ML detection performed in the 4×4 or 8×8 system with 4-QAM constellation and ZF-based ordering. The result shows that 8×8 system requires more time to be allocated on the combined block ML detection, especially in lower SNR area.

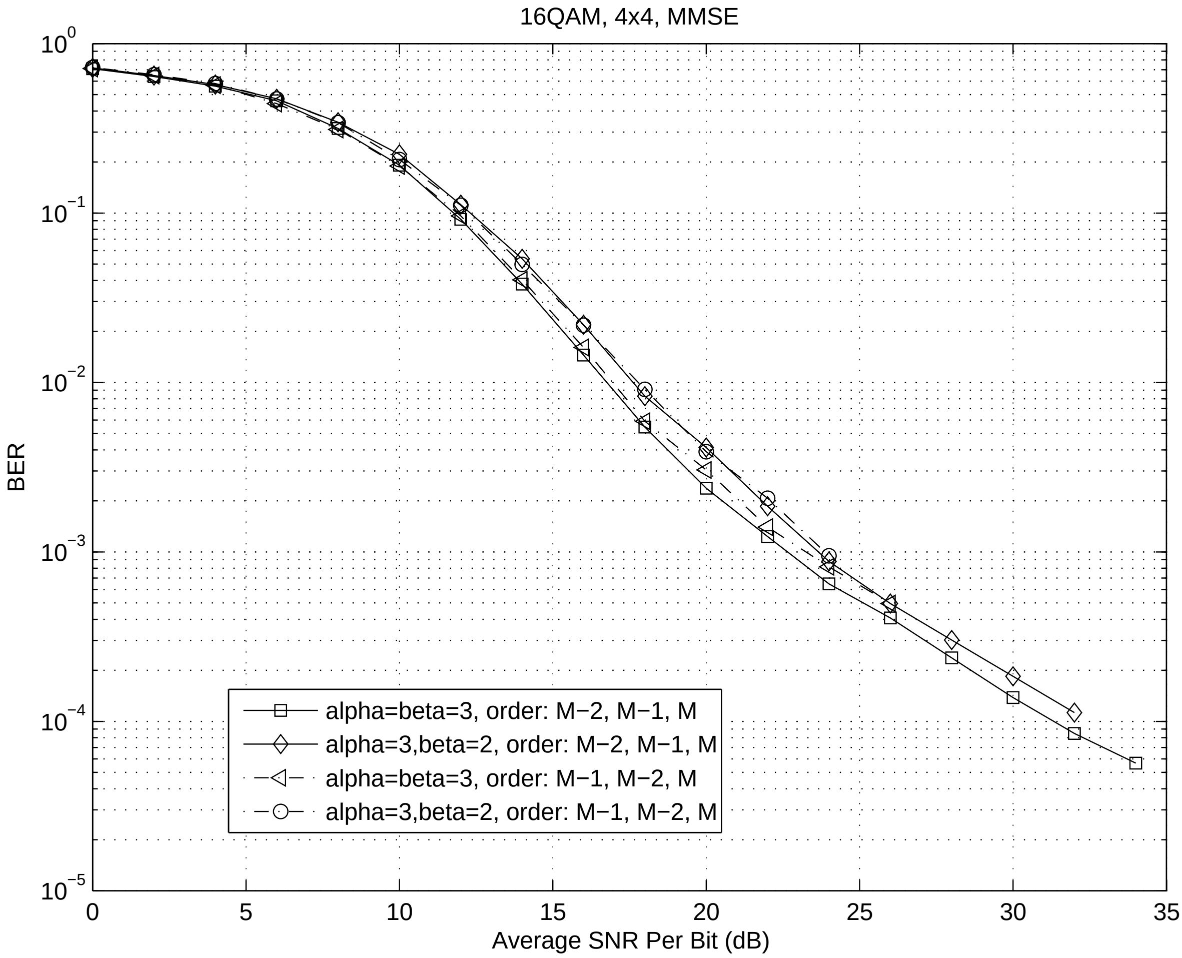

Figure 5.6 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE, Different detection order

For high SNR area, comparison results achieve agreement in most of times. Therefore, very few time is needed for block ML detection and there is no big difference between the required time for block ML detection of the two models.

Fig. 5.9 depicts the percentage of block ML detection performed in the proposed algorithms, with the same setup of Fig. 5.6. The performance with the comparison order of $\bar{x}_{M-2}$, $\bar{x}_{M-1}$ and $\bar{x}_M$ requires more time to be allocated on the block ML detection under the same α and system setup than that of $\bar{x}_{M-1}$, $\bar{x}_{M-2}$ and $\bar{x}_M$. That can be used to explain why the previous one has a better BER performance.

Fig. 5.10 depicts the percentage of block ML detection performed in the proposed algo-

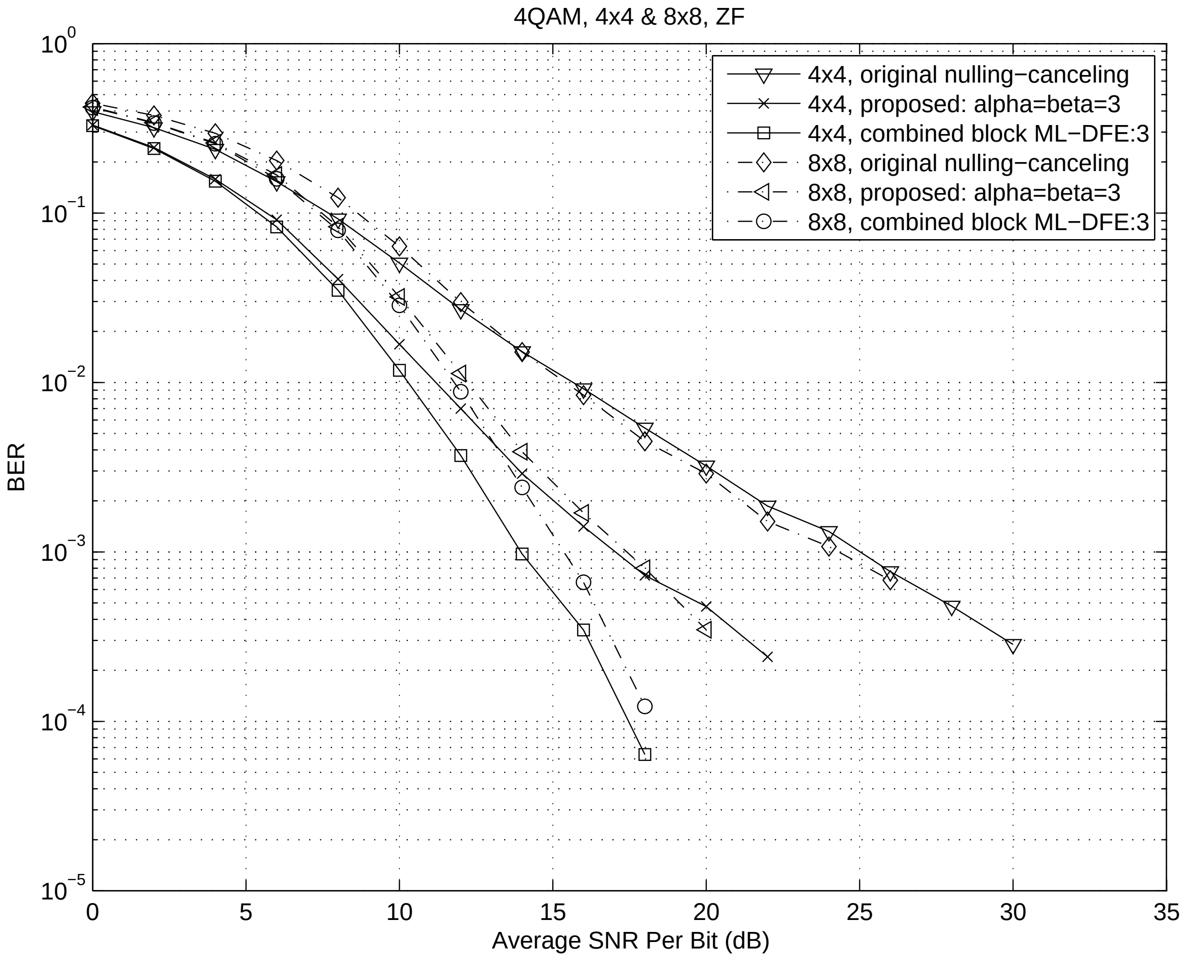

Figure 5.7 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$ or $M = N = 8$; 4-QAM, ZF

rithms, for the same setup as in Fig. 5.5. The MMSE based SNR ordering requires less time in block ML at the same SNR level compared to the ZF based ordering, especially when SNR is low. For MMSE-based ordering, ϵ drops below 0.1 at about 10dB and 5dB for $\alpha = 3$ and $\alpha = 2$ respectively. However, for ZF-based ordering, ϵ drops below 0.1 at about 14dB and 6dB for $\alpha = 3$ and $\alpha = 2$ respectively. In both cases, as SNR increases, the percentage of ML-decoded blocks decreases. This is not surprising, because at high SNR, the BER is already low, so the chance for disagreement between two nulling-canceling detections will also be small.

Fig. 5.11 depicts the percentage of block ML detection of a 8×8 system with 4-QAM or 16-QAM constellation and $\alpha = 2$ or $\alpha = 3$ block ML antennas number. When choosing the

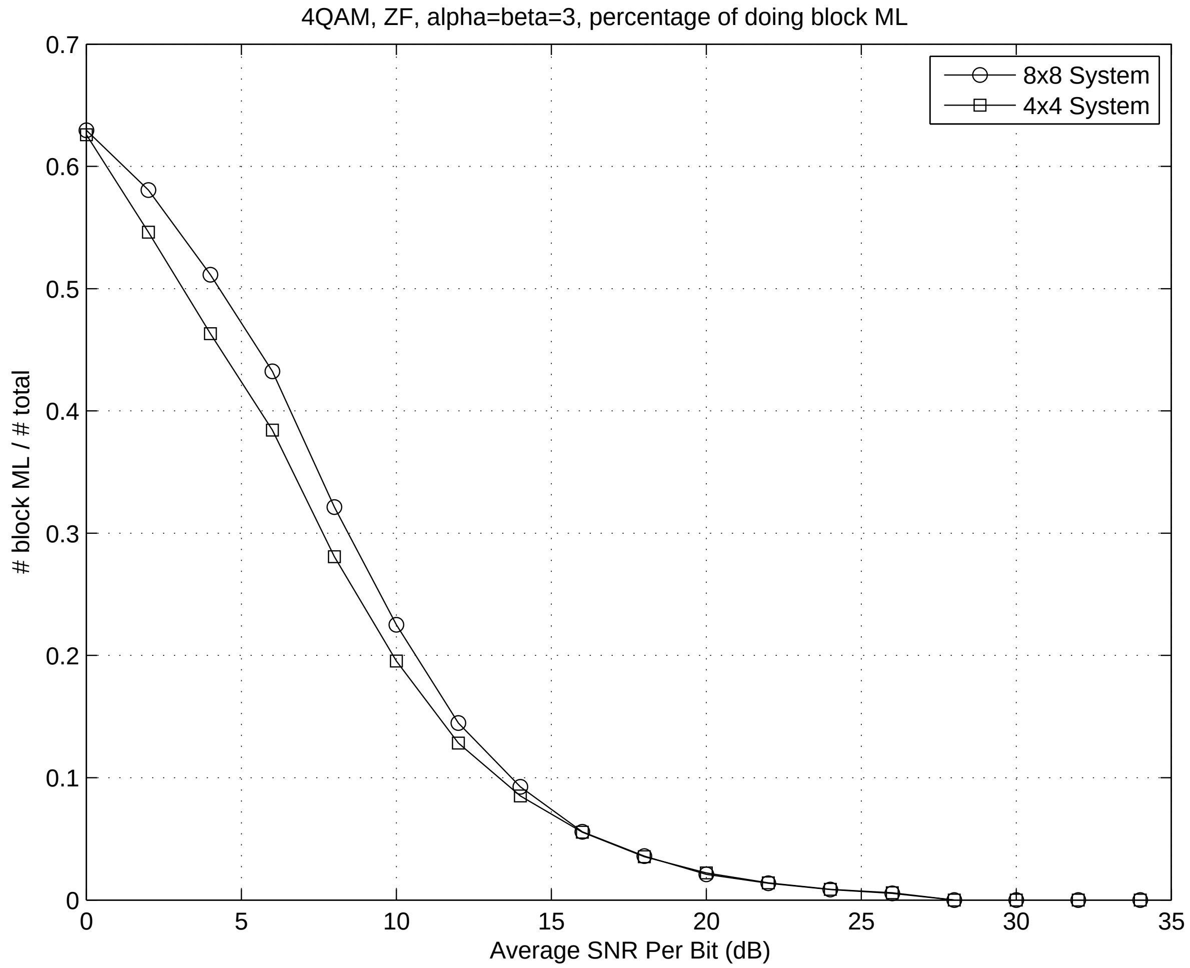

Figure 5.8 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$ or $M = N = 8$; 4-QAM, ZF

same antenna number for block ML detection, 16-QAM requires more time to be allocated on the block ML detection under the same system setup. Intuitively it is because 16-QAM constellation has a sophisticated constellation diagram than 4-QAM, which makes the symbol estimates of different orderings hard to achieve an agreement, therefore, requires more times on the block ML detection for better decoding results. For $\alpha = 2$, the complexity factor ϵ drops to less than 0.1 at 3dB for 4-QAM and at 13dB for 16-QAM. For $\alpha = 3$, the complexity factor ϵ drops to less than 0.1 at 7dB for 4-QAM and 16dB for 16-QAM.

Fig. 5.12 depicts the percentage of block ML detection performed in the proposed algorithms, for the same setup as in Fig. 5.1. ϵ does not monotonically drops when the SNR

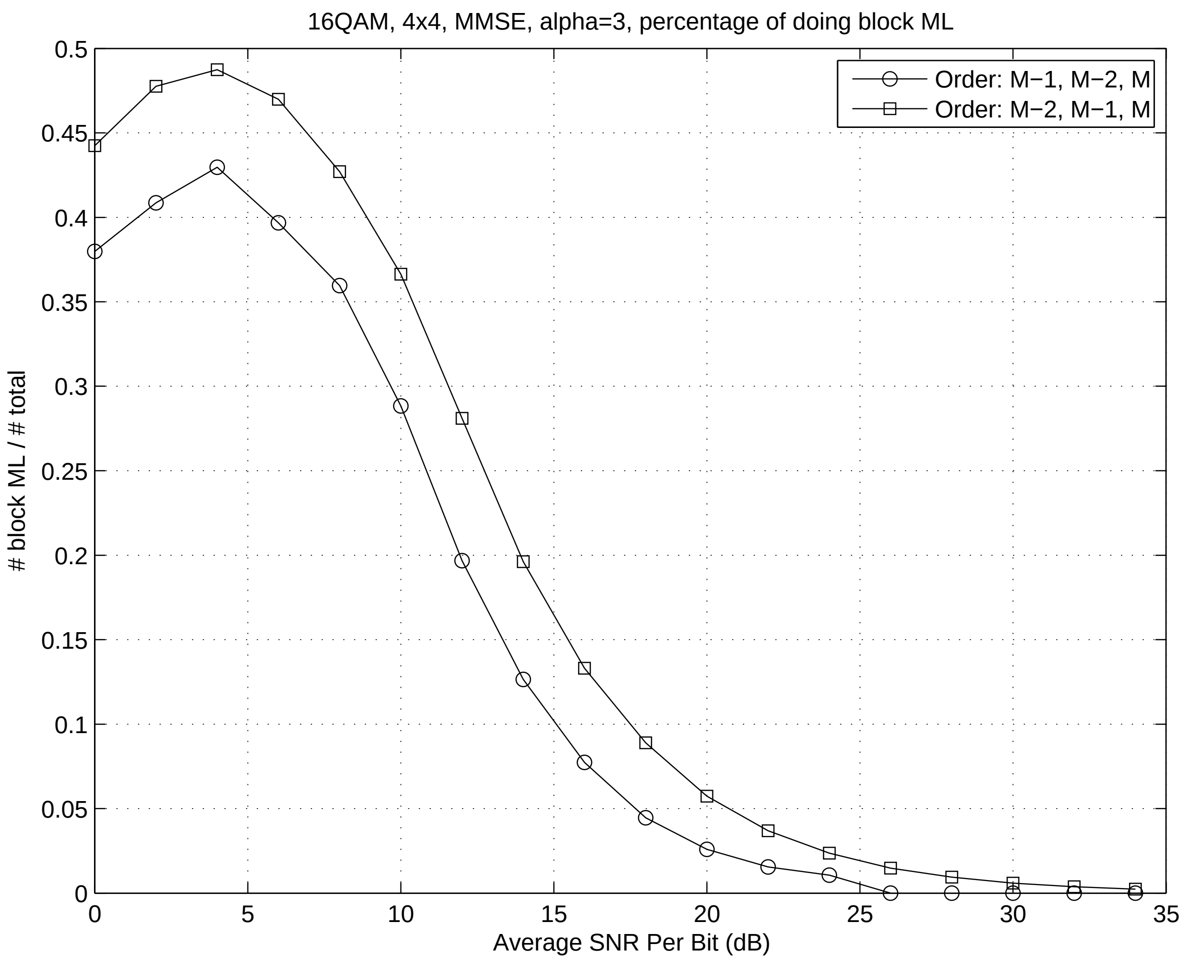

Figure 5.9 Complexity of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE, Different detection order

increase in this model. $\alpha = \beta = 3$ requires more time than $\alpha = \beta = 2$ on block ML detection at the same SNR level. For $\alpha = \beta = 2$, ϵ drops to less than 0.1 at 14dB and for $\alpha = \beta = 3$, ϵ drops to less than 0.1 at 18dB.

Fig. 5.13 depicts the BER performance of a 4×4 system with 16-QAM constellation and MMSE-based ordering, under original nulling-canceling, $\alpha = \beta = 2$, unordered compare with ordered and unordered compare with ordered one with the last 2 columns changed (both for $\alpha = \beta = 2$). The model is depicted in the last part of Chapter 3.3. The performance of the two unordered comparisons lie between the performance of the original nulling-canceling and

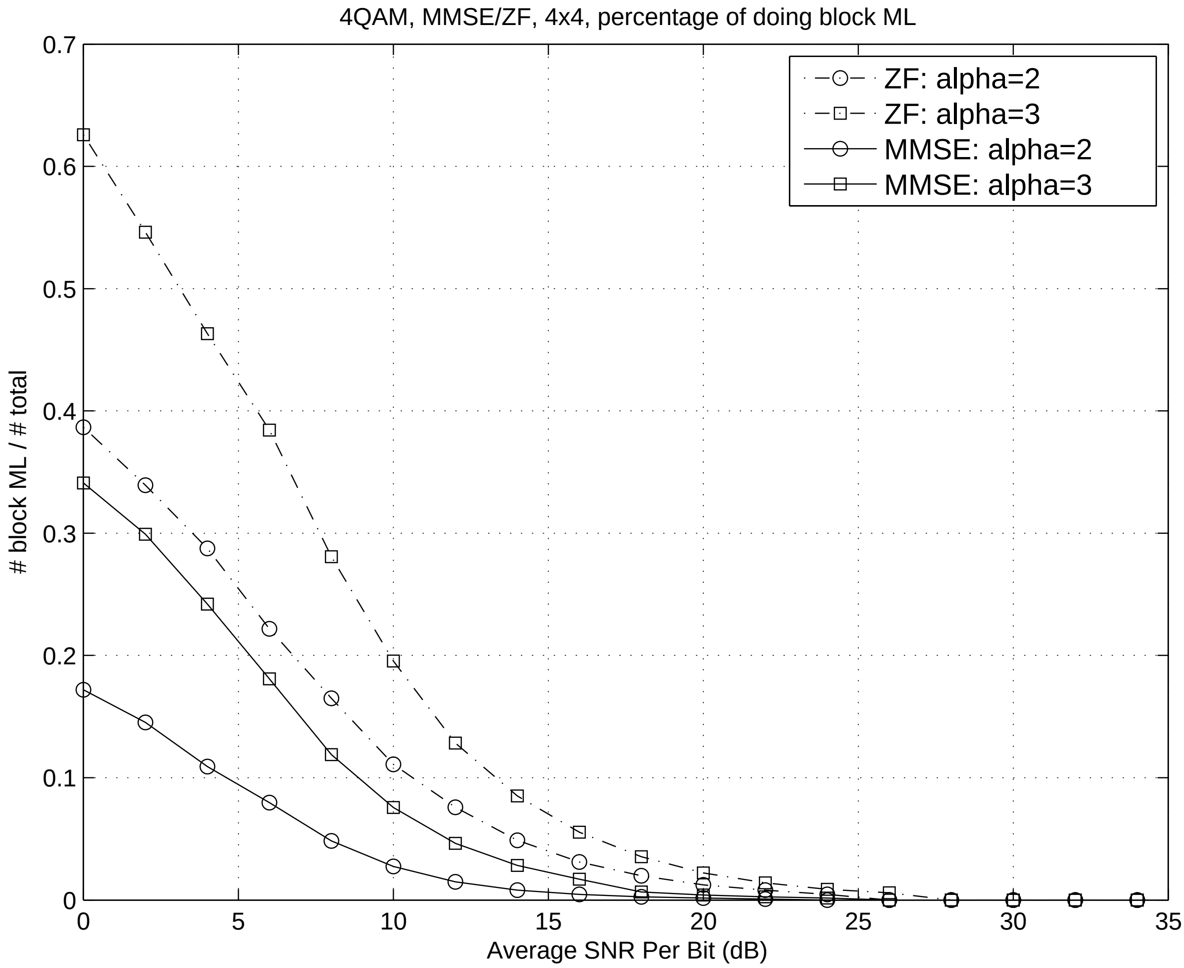

Figure 5.10 Complexity of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 4-QAM, MMSE & ZF

our proposed $\alpha = \beta = 2$ algorithms. The performance of the unordered with the no columns changed ordered one is better than the performance of the unordered with the last two columns changed one. The one with no columns changed depicts a performance which is close to the proposed $\alpha = \beta = 2$ algorithm.

Fig. 5.14 depicts the percentage of block ML detection performed in the algorithms in Chapter 3.3, for the same setup as in Fig. 5.13. Both of the algorithms that compare the unordered one with the ordered one require less time for block ML detection than the proposed $\alpha = \beta = 2$ algorithm for most SNRs. The unordered one compares with no columns changed one requires more time in block ML detection than the unordered one compares with columns

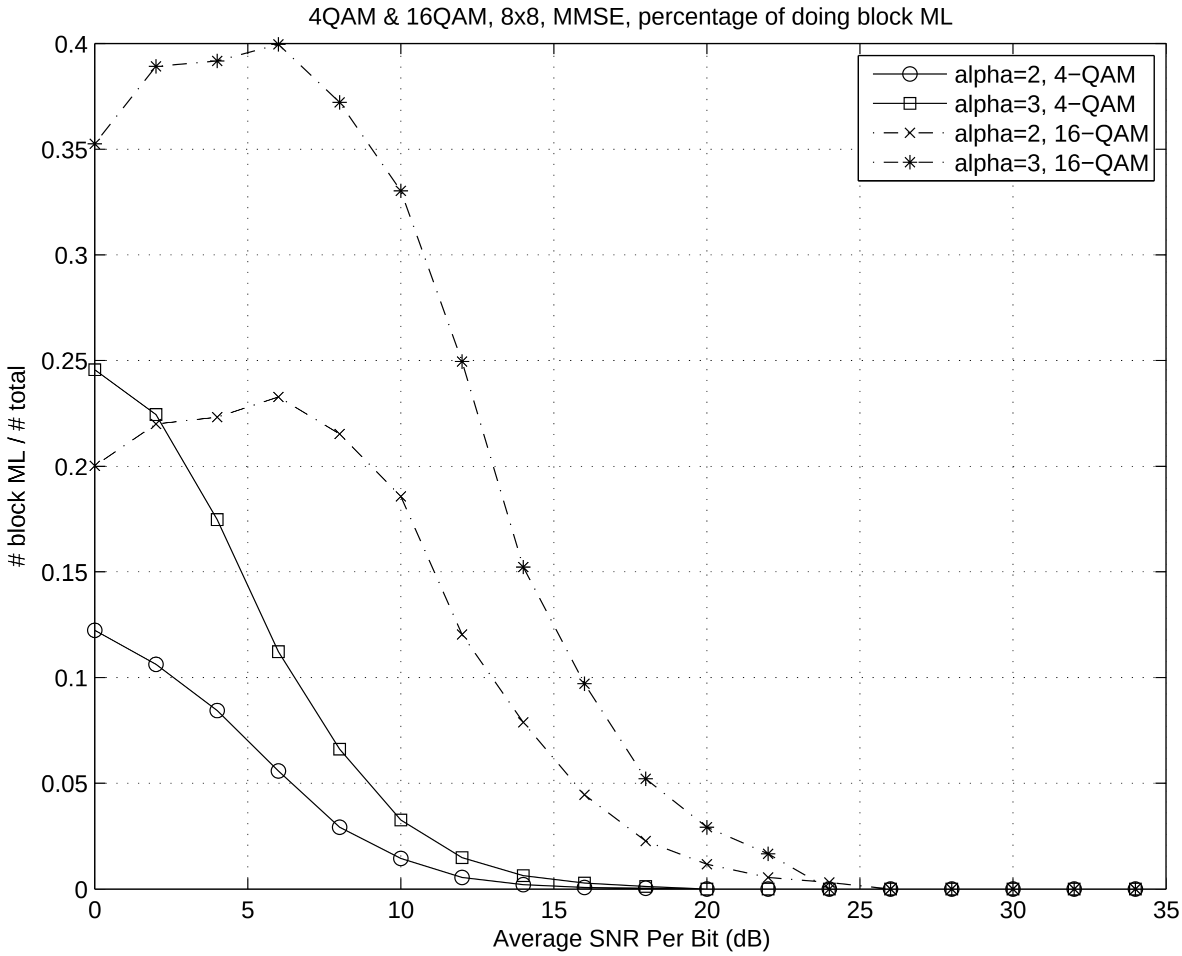

Figure 5.11 Complexity of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 8$; 4-QAM & 16-QAM, $\alpha = 2$ or $\alpha = 3$, MMSE

changed one, therefore achieves a better BER performance result.

Given the complexity restriction and the specific constellation model, we can always get the specific pair of α and β numbers which will satisfy our requirement and channel condition. This is really useful for the implementation of real wireless communication systems with specific channel and environment constraint.

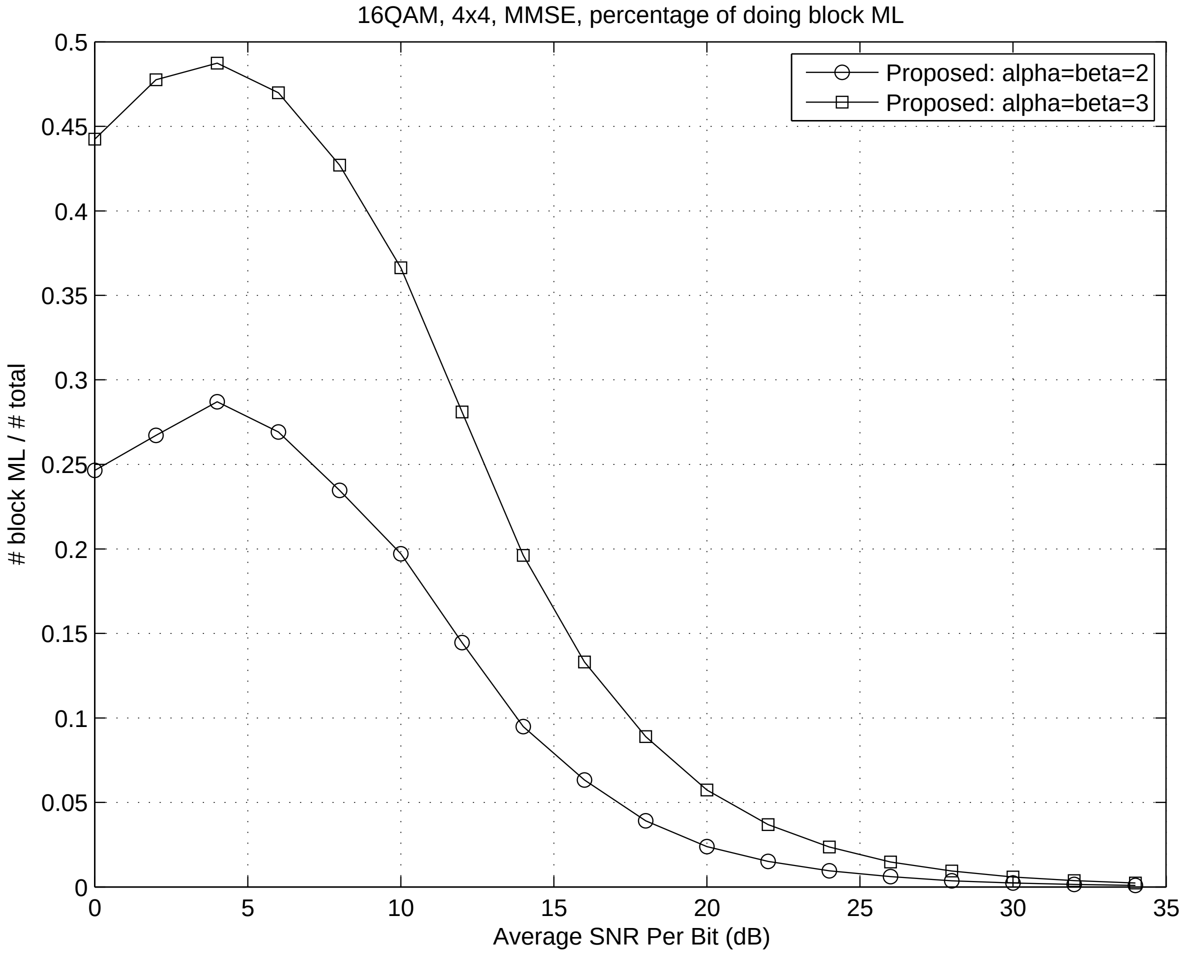

Figure 5.12 Complexity of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE

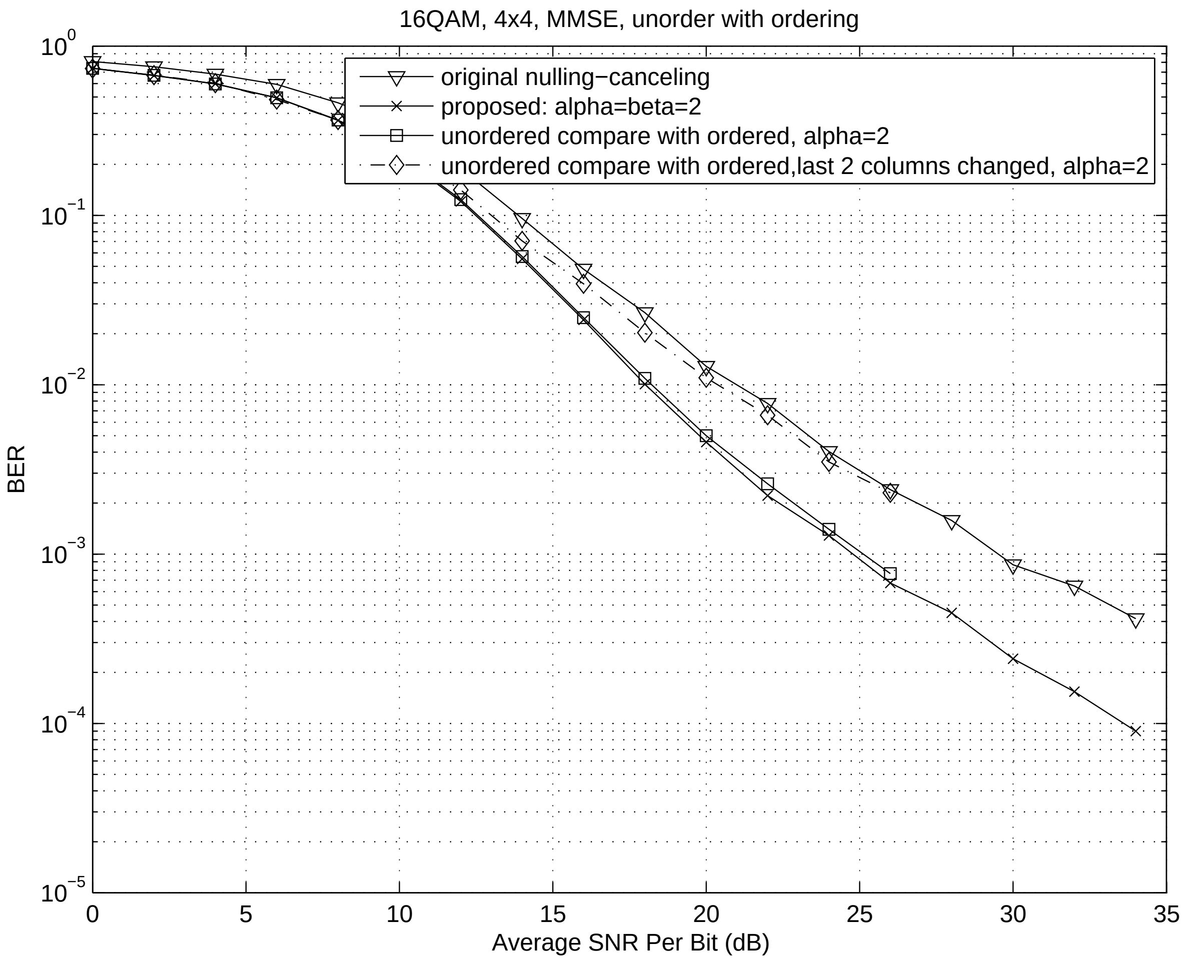

Figure 5.13 BER of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE, Unorder with ordering

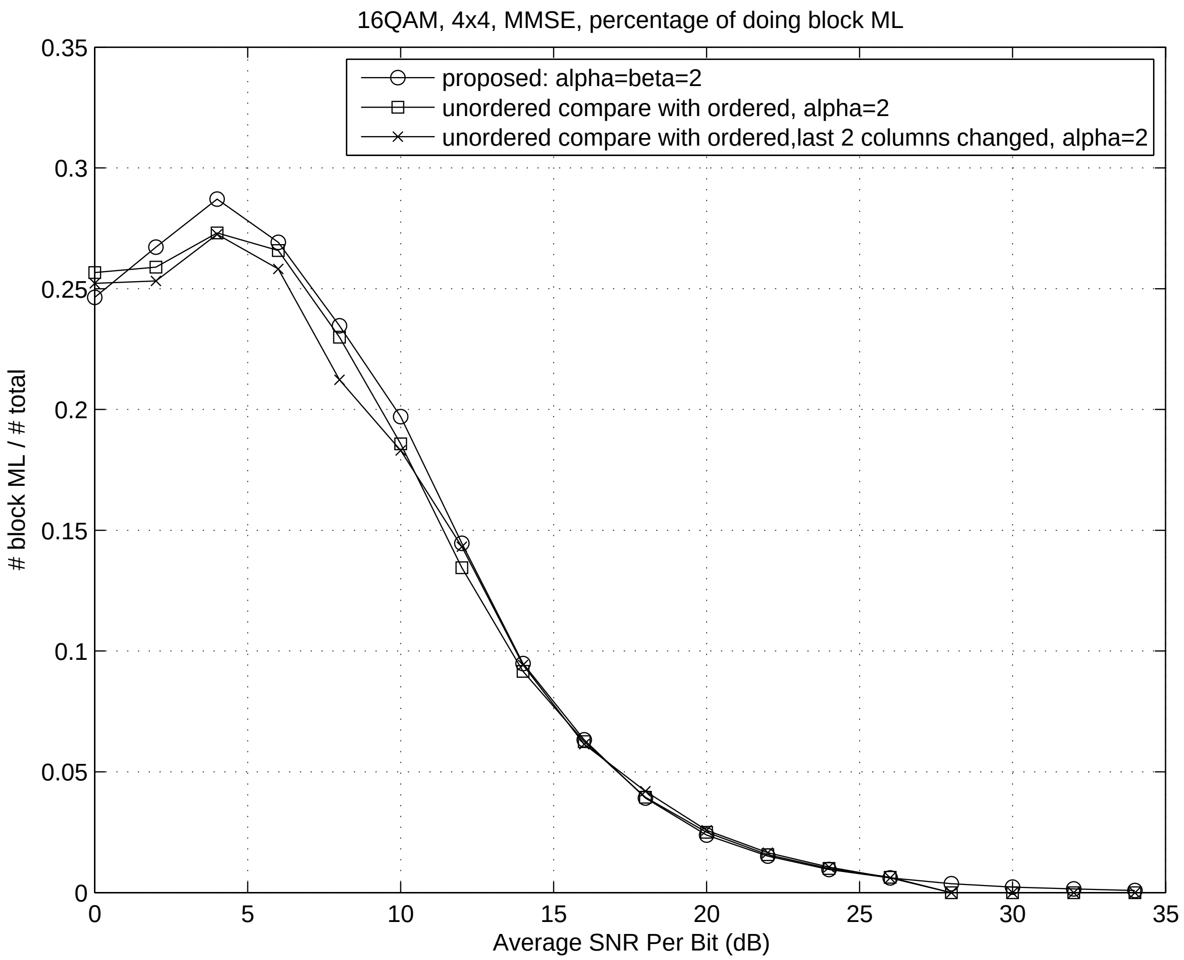

Figure 5.14 Complexity of decoding algorithms versus E_b/N_0 per transmit antenna, $M = N = 4$; 16-QAM, MMSE, Unorder with ordering

CHAPTER 6. CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

In this thesis, we proposed an MIMO detection algorithm that combines the nulling and canceling algorithm and block maximum likelihood detection algorithm to achieve improved performance with reasonable computational complexity. The algorithm relies on comparing the detection results of two nulling-canceling algorithms with different orderings. Existing decoding algorithms are summarized with the advantages and disadvantages given. Complexity analysis and BER analysis are given and summarized in this thesis. Simulation results show that the BER performance of the V-BLAST system can be improved by adopting our proposed detection technique. ZF-based ordering can achieve a better performance gain at the same system setup and SNR level compares to MMSE-based ordering. Time required for block maximum-likelihood decreases when SNR increases for some cases, and approaches zero at high SNR level, which makes the algorithm efficient. The decoding order of the nulling-canceling algorithm is discussed with the simulation of our proposed method. In our proposed algorithm, the decoding complexity increase is small at high SNR because the comparison results in agreement most of times, reducing the need for block ML detection. The BER performances of an unordered system with BPSK or 4-QAM modulation and hybrid detection algorithms are given, under the joint consideration of nulling-canceling α subchannels and block maximum-likelihood detection of β subchannels.

6.2 Future Work

In this study, we implement our system with 4-QAM and 16-QAM constellation diagram. It is also interesting to consider other constellation schemes to evaluate the performance of the systems. Cross-layer design can be considered in our proposed algorithm in the future, which combines the nulling-canceling algorithm and maximum-likelihood algorithm with considering of energy constraint, delay, transmitter power allocation and throughput node lifetime optimization. Our proposed selective ML algorithm can also be applied on the Log-Likelihood Ratio (LLR) based ordering scheme. A performance comparison can be made between our proposed method and sphere decoding algorithm also.

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